STATIC AND DYNAMIC BUCKLING OF LAMINATED COMPOSITE SHELLS

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ABSTRACT

This paper presents the results from numerical investigation of the behavior of cylindrical laminated shells subjected to suddenly applied loading. A numerical approach is used combining the finite element method with two different stability criteria namely the Budiansky-Roth and the phase-plane buckling criteria. In the finite element analysis an explicit time integration scheme is used implemented in the LS-DYNA commercial code. The response of the composite shells has been investigated at the presence and absence of static preloading. Results are presented for different values of the shell length, imperfection factors, stacking sequence, and laminae orientation, and for different static preloading magnitudes. Based on the presented results important conclusions can be drawn concerning the shell behavior and its sensitivity to different system parameter variations.

INTRODUCTION

Cylindrical shells have been extensively used in all types of structures. They are subjected to various loading both static and dynamic in nature. Among the most common types of loading are the axial compression and lateral pressure, which not only challenge the strength of the structure but could also cause deformations of unacceptably large amplitudes and could lead to loss of stability and collapse of the whole structure. Due to their numerous advantages composite materials are increasingly used in shell structures design. Therefore, the problems of investigating the behavior of laminated cylindrical shells subjected to different static and dynamic loading have drawn considerable attention of scientists, researchers, and designers in the last several decades. In the reviews of Svalbonas and Kalnins (Svalbonas, 1977), Hsu (Hsu, 1974), and Simitses (Simitses, 1987) different aspects and phenomena including such related to the dynamic stability of cylindrical shells are discussed. Volmir (Volmir, 1958) who utilized Galerkin's method first attempted the problem of dynamic buckling of axially loaded shells. Coppa and Nash (Coppa, 1964), and Roth and Klosner (Roth, 1964) applied the potential energy method to study this problem. Tamura and Babcock (Tamura, 1975) investigated the dynamic buckling of cylindrical shells with geometric imperfections applying the Budiansky-Roth criterion (Budiansky, 1962). The dynamic stability of suddenly loaded laminated cylindrical shells and the effect of static preloading upon the dynamic critical load was studied by Simitses, (Simitses, 1983 and 1989). Most of the above works investigated the shell structure behavior at axial compression. Huyan and Simitses (Huyan, 1997) considered the problem of dynamic buckling of geometrically imperfect cylindrical shells under bending moments, and Shaw et al. (Shaw, 1993) – under torsional loading. There are also works regarding the dynamic buckling of cylindrical shells subjected to lateral pressure: Al-Hassani et al. (Al-Hassani, 1970) presented and compared theoretical and experimental results for thin-walled metal tubes under a pressure pulse. Mustafa et al. (Mustafa, 1993) studied the dynamic buckling response of tubes immersed in water and subjected to external pressure.

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Schokker et al. (Schokker, 1996) investigated the dynamic instability of interior ring stiffened composite shells under hydrostatic pressure, and Gu et al.(Gu, 1996) studied the dynamic plastic buckling of cylindrical shells under general external impulsive loading using the energy criterion. Along with the theoretical work carried out in this area there are results of experimental tests and numerical simulations on metal and composite shells. Lindberg and Florence (Lindberg, 1987) published summarized results from research on buckling of metal cylinders under different types of loading. Kirkpatrick et al. (Kirkpatrick, 1988 and 1989) performed experimental and numerical studies on aluminum cylindrical shells subjected to impulsive external loading and studied the imperfection sensitivity varying the imperfections of the geometry and loading. Pegg (Pegg, 1991) investigated numerically the dynamic pulse buckling of cylinders of infinite length and different radius-to-thickness ratios. He also examined the effect of the pulse duration and the combination of static and impulsive dynamic loading (Pegg, 1994). Mustafa et al. (Mustafa, 1993) used a finite element numerical approach for immersed tubes subjected to an external pressure pulse and compared their results with previously published experimental and theoretical results for the same model. Shaw et al. (Shaw, 1993) presented results from numerical solutions of fiber-reinforced composite cylinders subjected to axial and torsional impulsive loads. To assess the dynamic stability of the laminated shells they implemented two different buckling criteria, namely the Simitses, and the Budiansky-Roth criteria. Gilat and Aboudi (Gilat, 1995) published numerical results illustrating the effect of nonlinear behavior on the dynamic response of composite plates and shells. Huyan and Simitses (Huyan, 1997) investigated numerically the dynamic stability of imperfect laminated cylinders under axial and bending loading. They discussed the effect of the load duration and the imperfection amplitude on the critical loading. Schokker et al. (Schokker, 1996) published results from extensive numerical tests on unstiffened and ring stiffened shells under hydrostatic pressure loading. They presented data for various parameter variations for isotropic and anisotropic shells with different lamina orientation.

This paper presents the results from numerical investigation of the behavior of cylindrical laminated shells subjected to suddenly applied lateral pressure. A numerical approach is used combining the finite element method with two different stability criteria: the Budiansky-Roth and the phase-plane buckling criteria. In the finite element analysis an explicit time integration scheme is used. The response of the composite shells has been investigated at the presence and absence of static preloading. Results are presented for different values of the shell length, imperfection factors, stacking sequence, and laminae orientation, and for different static preloading magnitudes. The herein investigated case of suddenly applied lateral pressure with static preloading is of particular importance because it is a representation of numerous practical problems: submarines subjected to underwater explosion; airplanes or fuel tanks under gust loading; submerged pipelines under impact loading; jet engine casings under rotor unbalance to name just a few.

STATEMENT OF THE PROBLEM

A cylindrical laminated shell, fig. 1.a, of length L, radius R, and total wall thickness h_t is subjected to uniformly distributed lateral pressure p which is suddenly applied. The solutions to two basic problems are sought: first – the critical pressure p_{cr}, which causes buckling of the shell is defined for different loading time durations starting from very small time duration to practically infinity without static preloading, fig. 1.b; second – in the presence of previously applied static preloading, values of p_{cr} for infinite duration are sought for different values of the statically applied preloading, fig 1.c. Imperfections in the geometry of the shell are introduced to initiate buckling. The above described problems are solved for different shell geometries (L/R and R/h_t ratios), different lamina stacking sequences, and different geometric imperfections. Thus the relevant response relationships are acquired.

GEOMETRY, PROPERTIES AND LOADING

The following parameter values were chosen as a basis for the investigation: cylinder radius $R = 7$ *in*. (17.78 *cm*), cylinder length $L = 14$ *in*. (35.56 *cm*), imperfection factor $\xi = 0.01$, shell total thickness $h_t = 0.14$ *in*. (0.3556 *cm*), and laminae stacking – $[90^\circ_5/0^\circ_5]_S$. Throughout the analysis the cylinder radius was kept constant and the other parameters were varied to assess their variation influence on the shell behavior. To get the system response to one parameter variation, that parameter was varied and all the rest were kept at the above stated values. Due to the symmetry only half of the shell along the cylinder axis was discretized as shown in fig. 2. The boundary conditions are as follow: symmetric at the symmetry midsection plane – no axial displacements, and no rotations around the radial axis and the circumference tangent. At the cylinder end there are no radial and circumferential displacements, and in the axial direction all end points have the same displacements, thus simulating the presence of a rigid endcap. The finite element mesh had 48 elements around the shell circumference and 8 elements along the cylinder length.

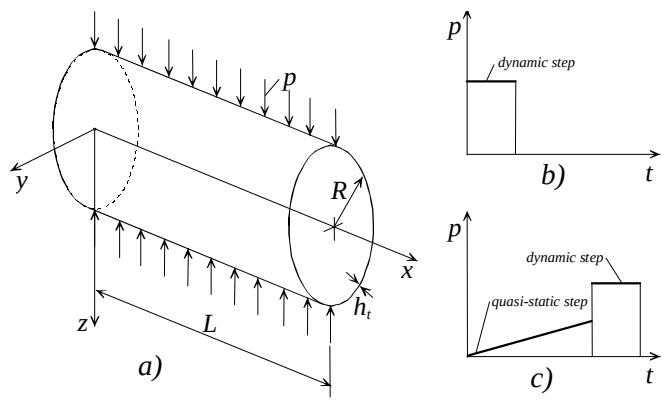

Fig. 1. Model Geometry and Loading

The shells are made of Graphite/Epoxy composite laminae. Each lamina has a thickness of 0.007 *in*. (0.1778 *mm*), and all laminae stacking sequences investigated form a cross-ply or an angle-ply symmetric laminate.

The laminae were assumed made of an elastic homogeneous orthotropic material with the following material properties: density $\rho = 1.497\times10^{-4}\ lbf{\cdot}s^2/in^4$ ($1.6\times10^3\ kg/m^3$); elastic moduli

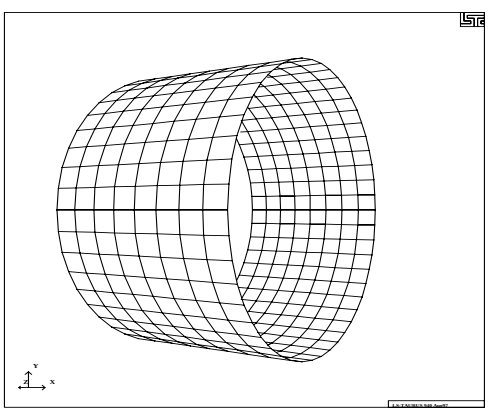

Fig. 2. Finite Element Model

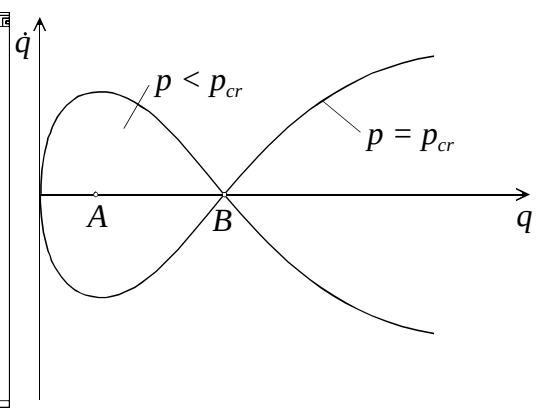

Fig. 3. Determining the Critical Loading by the Phase-Plane Criterion

$E_{11} = 0.1985\times10^8$ *psi* (136.9 *GPa*), $E_{22} = E_{33} = 0.143\times10^7$ *psi* (9.860 *GPa*); Poisson's ratios $\nu_{12} = \nu_{13} = 0.293$, $\nu_{23} = 0.45$; shear moduli $G_{12} = G_{13} = 0.82\times10^6$ *psi* (5.654 *GPa*), $G_{23} = 0.39\times10^6$ *psi* (2.689 *GPa*). Here, the subscripts of the engineering constants denote the corresponding direction: 1 – the fiber direction; 2 – the direction perpendicular to the fibers in the lamina plane; 3 – normal to the lamina plane.
Initial geometric imperfections are introduced in the model to trigger the buckling mode. The imperfect geometry is produced by combining several eigenmodes of the perfect shell. The imperfections are evaluated through the value of an imperfection factor ξ. Investigations are made for three different values of the imperfection factor ξ: $\xi = 0.01, 0.05, 0.1$. For the basic values of the rest of the model parameters, $\xi = 0.01$ results in a maximum radial displacement of the midsection nodes, ${}_{\max}w_{imp}$, equal to 0.02465 times the shell thickness h_t $\{{}_{\max}w_{imp} = 0.02465\times h_t = 0.00345$ *in*. (0.08763 *mm*)$\}$. For the same model $\xi = 0.05$ results in ${}_{\max}w_{imp} = 0.01232\times h_t = 0.01725$ *in*. (0.43815 *mm*), and $\xi = 0.1$ results in ${}_{\max}w_{imp} = 0.2465\times h_t = 0.0345$ *in* (0.8763 *mm*).

Two types of analyses were performed: a dynamic analysis with suddenly applied lateral pressure with no prior loading present, and a dynamic analysis with suddenly applied lateral pressure at the presence of previously applied static pressure.

SOLUTION APPROACH

The problems were numerically solved using the LS-DYNA explicit finite element code. Recent studies (Tabiei, 1998) showed that due to its advantages for this type of problem the explicit time integration proved more efficient than the implicit time integration scheme. When assessing the stability of the shells two buckling criteria were used:

The Equation of Motion Approach - Budiansky-Roth criterion (Budiansky, 1962): The equations of motion are solved for various values of the loading and the value at which there is a significant jump in the response is assumed critical. When monitoring the system response through displacements of selected points for small values of the loading parameter, small oscillations are observed, the amplitudes of which gradually increases as the loading is increased. When the loading reaches its critical value, the maximum amplitude experiences a large jump. Therefore implementation of this criterion requires to solve the equations of motion for different values of the loading parameter, then plot the displacement amplitude versus loading curve from which the critical loading value is determined (see fig. 5).

The Phase-plane Approach: If q is a parameter used to monitor the system response (usually a typical displacement), the phase-plane is the plane in which the phase trajectories (plots of $\dot{q} = dq/dt$ versus q) lie - fig. 3. For loads smaller than the critical, the system simply oscillates about the static equilibrium point *A*, and at loading equal or greater than the critical escaping motion, indicating buckling, occurs through the unstable static equilibrium point *B*.

Figures 4 through 9 show results from the problem acquired by combining both buckling criteria with the explicit finite element analysis. As seen from fig. 4 when reaching the critical pressure value, for a small increase in the applied lateral pressure we witness an abrupt change in the system response. The phase-plane curves, figs. 8 and 9., also indicate that the critical loading has been reached. Several more iterations on the applied pressure with values below and above the critical provide the information for the maximum amplitude versus pressure curve, fig. 5, from which the same value for the critical pressure is derived.

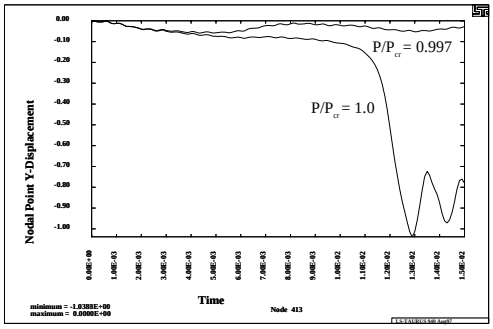

Fig.4.Displacement Curves for Loading Values Below and Equal to the Critical

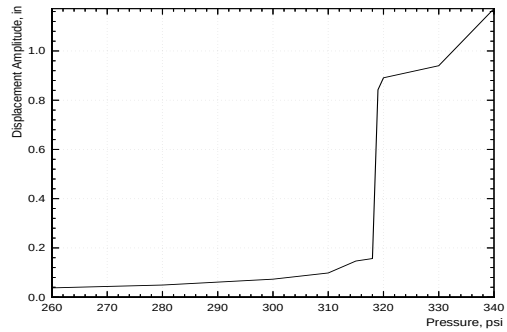

Fig.5. The Budiansky-Roth Criterion Plot using the Displacement Amplitudes

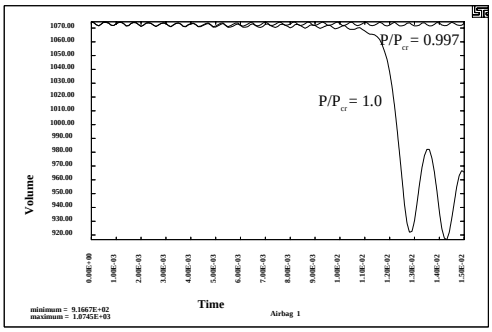

Fig.6. Shell Volume Curves for Loading Values Below and Equal to the Critical

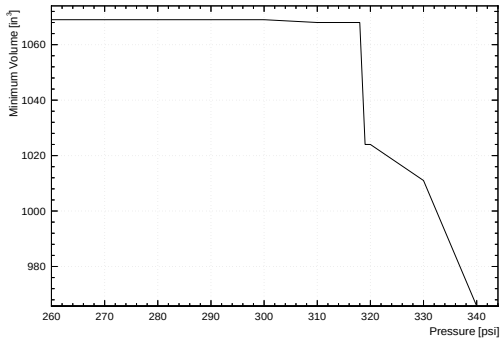

Fig.7. The Budiansky-Roth Criterion Plot using the Shell Volume

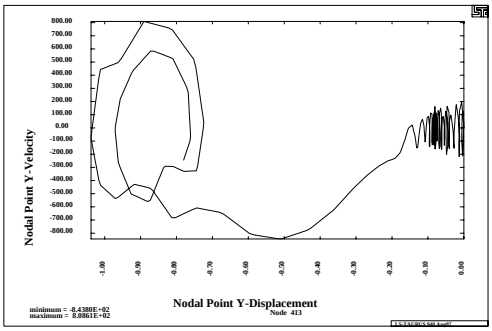

Fig.8. Phase Trajectory for P/P$_{cr}$ = 1.0

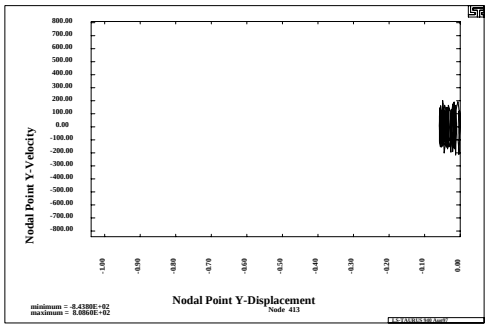

Fig.9. Phase Trajectory for P/P$_{cr}$ = 0.997

Note that to be able to get good results from the analyses, the points for which displacements are to be monitored are to be carefully chosen, otherwise the plots produced may be rather obscure and confusing. In our case we monitored the radial displacements of several points from the midsection of the cylinder, where the largest displacement values were expected. Examples with randomly chosen points proved that the proper choice is essential for the results to be adequate. Therefore, since dealing with a closed volume structure, we can use the volume to monitor its response. Since the volume change is related to the displacements of all nodes it can be used as an overall estimate for the shell behavior instead of the nodal displacements. As seen from figures 6 and 7 the shell volume can be successfully used to estimate the dynamic buckling pressure using the Budiansky-Roth criterion.

When investigating the effect of static preloading, the analysis was divided into two steps in the finite element analysis – a quasi-static step in which the preloading was applied, and a

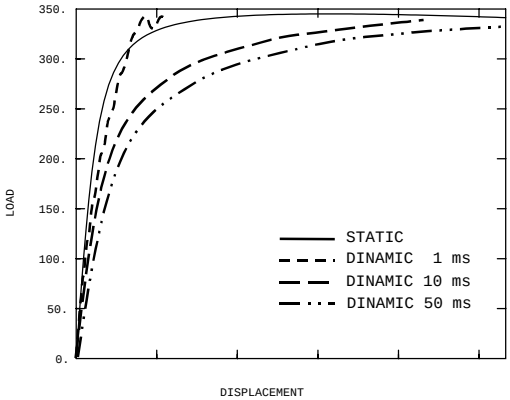

Fig.10.Quasi-Static and Static Analyses

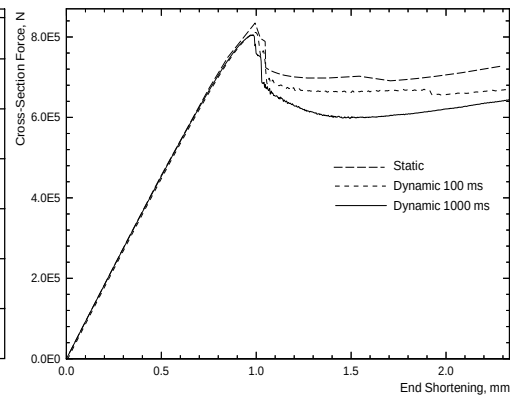

Fig.11. Buckling of Axially Loaded Cylinder

second dynamic step at the beginning of which the additional dynamic loading is suddenly applied (see fig. 1.c). Since the explicit analysis is a dynamic analysis, to model the static preloading we applied the static pressure gradually, with small enough rate as to keep the inertia effects negligible, thus emulating a static analysis. Experiments were made with different rates of increase of the preloading to be able to choose an appropriate duration for the quasi-static step. Some results are shown on fig. 10 where quasi-static steps of different duration are compared with the result from a static finite element code. Fig. 11 shows similar results from static buckling of an axially loaded cylinder. The problem was solved with a static finite element code and with an explicit dynamic code for two different loading rates. As we see the critical pressure is basically the same. There are differences in the post-buckling behavior, but there the dynamic analysis is likely to produce more realistic results having in mind the dynamic nature of the post-buckling behavior.

RESULTS AND DISCUSSION

The numerical results were acquired using the LS-DYNA finite element code. To assess the shell behavior the displacements of some midsection points in the transverse plane were traced. The variation of the cylinder volume throughout the analysis was also traced by defining an airbag within the cylinder and tracing its volume. To find the critical value for the suddenly applied pressure, for different load durations, iterations were performed on the pressure magnitude, as well as on its duration. The critical pressure for infinite time duration was assumed reached when no further change in the critical pressure was observed when increasing its duration. Thus the acquired values for the critical pressure and the

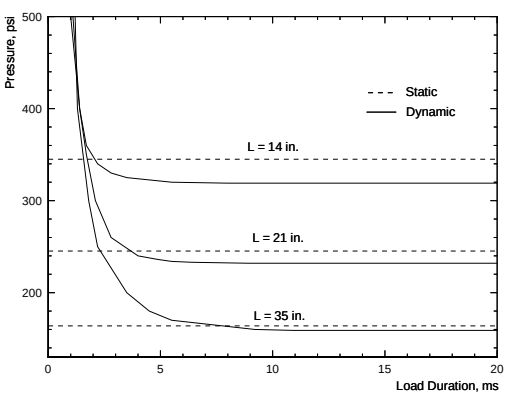

Fig.12. Static and Dynamic Critical Pressure for Different Cylinder Length, *L*

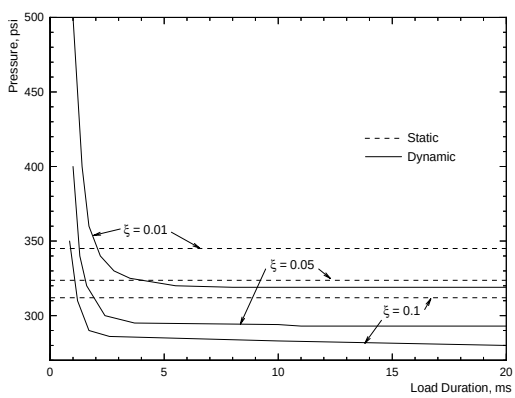

Fig.13. Static and Dynamic Critical Pressure for Different Imperfection Factor, ξ

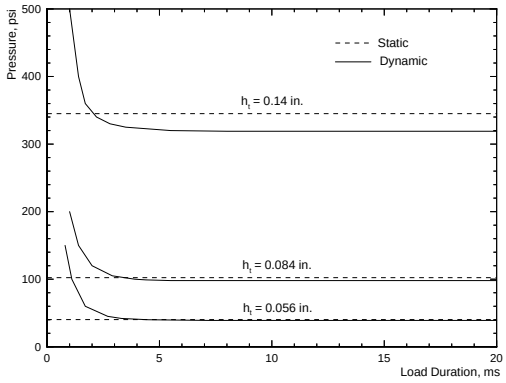

Fig.14. Static and Dynamic Critical Pressure for Different Shell Thickness, h_t

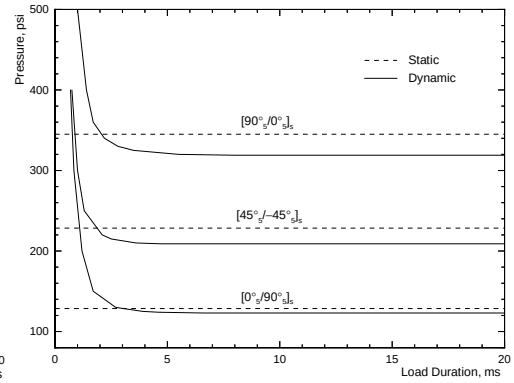

Fig.15. Static and Dynamic Critical Pressure for Different Stacking Sequence

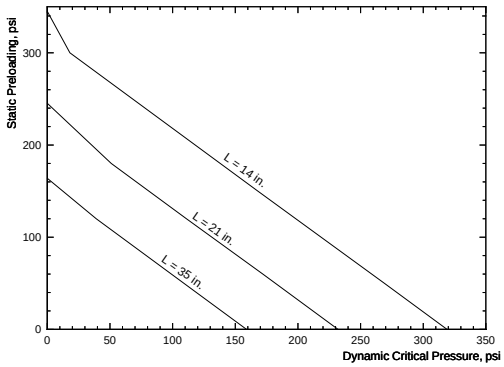

Fig.16. Static Preloading vs. Dynamic Critical Pressure for Different Cylinder Length, L

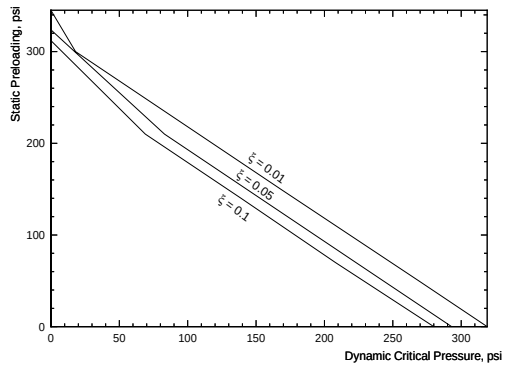

Fig.17. Static Preloading vs. Dynamic Critical Pressure for Different Imperfection Factor, ξ

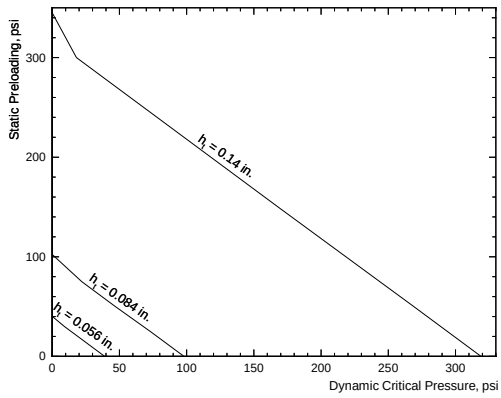

Fig.18.Static Preloading vs. Dynamic Critical Pressure for Different Shell Thickness, h_t

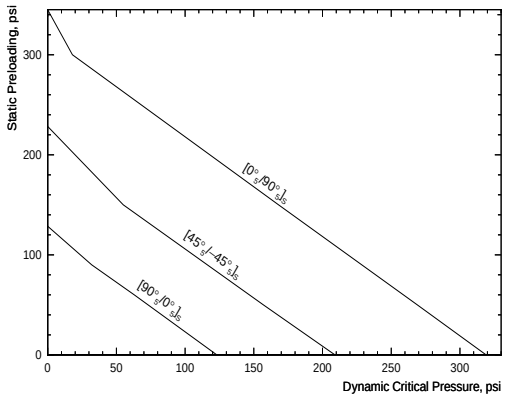

Fig.12.Static Preloading vs. Dynamic Critical Pressure for Different Stacking Sequence

corresponding time durations were plotted to form the critical pressure versus time duration curves. Fig. 12 shows these plots for three different values of the cylinder length L = 14, 21, and 35 *in*. (L/R = 2, 3, and 5). Fig. 13 shows the plots for different imperfection factors ξ = 0.01, 0.05, and 0.1 (all the other parameters have their basic values). Fig. 14 shows results for different shell thickness, h_t = 0.14, 0.084, and 0.056 *in*. (0.3556, 0.2134, and 0.1422 *cm*). The different shell thickness was realized by changing the number of shell layers, keeping the

thickness of each layer constant and equal to 0.007 *in*. (0.178 *mm*). Thus h_t = 0.14 *in*. corresponds to a stacking sequence $[90°_5/0°_5]_S$, h_t = 0.084 *in*. corresponds to a stacking sequence $[90°_3/0°_3]_S$, and h_t = 0.056 *in*. corresponds to $[90°_2/0°_2]_S$. Fig. 15 presents the critical pressure versus time duration curves for three different lamina orientations when forming the shell laminate. In figures 12-15 the static critical loads are also presented for the various constructions. Figures 16 through 19 show how the dynamic critical pressure varies when changing the value of the static preloading and performing the same parameter variations as conducted for the suddenly applied pressure with no prior loading.

Figures 12-19 show how the different parameters investigated affect the system behavior. Different L/R, R/h_t, ξ, and stacking sequences are considered to determine their influence on the critical dynamic pressure. From the generated results the following can be observed:

- As the time duration of the applied dynamic pressure is increased the critical load approaches that of the infinite duration;

- The imperfection amplitude has virtually the same effect on static critical load as on the dynamic critical load of infinite duration;

- For thin shells the static critical load is virtually the same as that of infinite duration. For this case the inertial effect is insignificant, however we can observe some variation for the case of moderately thick shells;

- For longer shells the static critical load is almost the same as that of infinite duration.

CONCLUSIONS

The present study demonstrates that when investigating the dynamic buckling of cylindrical laminated shells the explicit finite element approach is very attractive resulting in a very good accuracy along with its excellent computational efficiency. The study also implements two different dynamic buckling criteria and compares their performance and results. The results herein presented illustrate the stability of cylindrical laminated shells subjected to suddenly applied uniform lateral pressure and static preloading. The response sensitivity to different parameter variations investigated in the study could be used in laminated shell structures design in Industry and Defense.

ACKNOWLEDGMENTS

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