Problem 1. Given a linear, time-invariant system with the transfer function:

\[ T(s) = \frac{1}{s^3 + 8s^2 + 13s + 6} \]

a. Construct the controllable canonical form state space realization.

b. Construct the observable canonical form state space realization.

Problem 2. Given a linear time-invariant system described by the differential equations:

\[ \begin{align*}
\dot{y}_1 + 3 \dot{y}_1 + 2(y_1 - y_2) &= u_1 + \dot{u}_2 \\
\dot{y}_2 + 3(y_2 - y_1) &= u_2 + 2 \dot{u}_1
\end{align*} \]

Find a state space representation. Notice that derivatives of the inputs are present!

Problem 3. Consider the system described by the linear difference equations:

\[ \begin{align*}
y_1(k+2) + 10y_1(k+1) - y_2(k+1) + 3y_1(k) + 2y_2(k) &= u_1(k) + 2u_1(k+1) \\
y_2(k+1) + 4[y_2(k) - y_1(k)] &= 2u_2(k) - u_1(k)
\end{align*} \]

This is a discrete time system with two inputs and two outputs. Find the pulse transfer function relating each input to each output (there will be 4 of these) and then develop a set of state equations for the model. (Controllable canonical form is preferable, but not required.)