EXTENDED ABSTRACT

Genetic Optimization of Turbine Cascade Navier-Stokes Solutions

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Abstract

A new methodology has been developed to find the optimal aerodynamic turning of a turbine cascade. A Navier-Stokes algorithm and a genetic algorithm are linked within an automated design loop. The objective function has been either based on the lift-to-drag ratio or the lift coefficient. In our preliminary study presented in this extended abstract, the flow turning angle has been optimized by the robust genetic algorithm for given inlet flow properties and a fixed cascade thickness. The effects of uniform crossover, creep mutation, different random number seeds, population size and the number of children per pair of parents on the performance of the genetic algorithm have also been studied for this problem. It is shown that the optimum lift-to-drag ratio is achieved for lower flow turning where as a maximum lift requirement drives the aerodynamic design to higher flow turning angles.

The study has now focused on multi-objective genetic algorithm where blade thickness, blade pitch, inlet flow angle and flow turning angle are all varied.

1. Introduction

The numerical simulation of turbine cascades have been extensively treated over the last 20 years. Despite limitation due to modelling approximations such as turbulence and transition modelling, cooling and heat transfer calculations, these methods are now capable of analysing the performance of turbine blades with a flow accuracy which is acceptable for most engineering purposes. Computational fluid dynamics has also matured to the point at which it is widely used as a key tool for aerodynamic design. However for the final design process, most designers still adopt a ‘trial and error’ approach, analysing the current design, and modifying it in function of the computational results or the experimental data, according to empirical rules or to their own experience. Numerical optimization methods aims to shorten and simplify this iterative process, while significantly improving the design output. All optimization problems contains three components:

1) **Objectives** describing what one hopes to achieve through the optimization process. In this paper, this is the maximum lift and/or lift-to-drag ratio pertaining to turbine cascade.
2) **Design parameters** that describe how the system is to be adjusted in order to best meet the objectives. For example, parameters that determine the shape of the boundary. In this study, the design parameters are blade thickness, blade pitch, inlet flow angle and flow turning angle. The cascade geometry is modified accordingly.
3) **Constraints** that guides the optimization through states that must be satisfied. In our case, these are the non-linear set of differential Navier-Stokes equations and the associated inlet and exit boundary conditions.

Optimization techniques can be classified in three different categories: local, global or other methods. Local methods are gradient-based algorithms which only search part of the design space and stop after finding a local minimum. Adjoint, single or multi-grid preconditioners, alternating direction implicit methods are all local methods¹. Global methods are stochastic methods which take into consideration the entire search space. Genetic algorithms, simulated annealing, random search methods are all considered as global methods². They also have the advantage of operating on discontinuous design spaces. Other optimization algorithms that do not fall entirely within either of these two categories are one-shot or inverse methods³.

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The objective of the present paper is to use a two-dimensional Navier-Stokes solver together with a genetic optimisation algorithm. The 3rd order accurate flow algorithm has already been successfully used in the analysis of turbomachinery cascades. The present design aims to achieve a highly loaded and efficient turbine by initially starting from the Sanz subcritical turbine cascade.

2. The Navier-Stokes equations and their discretizations

The two-dimensional Navier-Stokes equations in conservation law form in a generalized body-conforming curvilinear coordinate system is used:

\[
\frac{\partial \hat{Q}}{\partial t} + \hat{E}_\xi + \hat{F}_\eta = \frac{1}{Re} \left( \frac{\partial \hat{P}}{\partial \xi} + \frac{\partial \hat{R}}{\partial \eta} \right) + \hat{T}
\]

where \( T \) is the body force term and the vector of conserved quantities and flux vectors are

\[
\hat{Q} = Q / J
\]

\[
\hat{E} = \frac{1}{J} (\xi Q + \xi E + \xi F)
\]

\[
\hat{F} = \frac{1}{J} (\eta Q + \eta E + \eta F)
\]

\[
\hat{P} = \frac{1}{J} (\xi P + \xi R)
\]

\[
\hat{R} = \frac{1}{J} (\eta P + \eta R)
\]

Re is the Reynolds number. Third order accurate TVD scheme of Yee has been adopted. This uses an upwind flux vector splitting differencing scheme for spatial discretization. The implicit LU-ADI algorithm of Beam and Warming has been used for time discretization.

\[
\left( I + h \delta_\xi \hat{A}^n - h \, Re^{-1} \delta_\xi \hat{L} \right) \left( I + h \delta_\eta \hat{B}^n - h \, Re^{-1} \delta_\eta \hat{M} \right) \Delta \hat{Q}^n
\]

\[
= -\Delta t \left( \delta_\xi \hat{E}^n + \delta_\eta \hat{F}^n - Re^{-1} \left( \delta_\xi \hat{P}^n + \delta_\eta \hat{R}^n \right) \right)
\]

where \( h \) is \( \Delta t \), \( \delta \) is the central differencing finite difference operator. The second term in the parenthesis of the left hand side is related with artificial viscosity. This term does not exist when the spatial discretization scheme is TVD.

Flow Boundary conditions
- Inflow: The inlet boundary condition specifies the Mach number and the stagnation enthalpy for each streamtube. Together with a given inlet geometry, this defines the mass passing through each streamtube. Moreover, the inlet streamtube slope is also specified.
- Outflow: For fully subsonic flows, no thermodynamic variables are specified at the exit. The outlet streamline slope is also not specified since this variable is already constrained by the Kutta condition, which is explicitly imposed as a constraint variable.
- Blade: The streamtube that defines the surface of the airfoil is considered to be an impermeable wall contour. Along this boundary, the solid wall condition simply states that the nodes along the pressure and suction side do not move.

3. The Genetic Algorithm (GA)
Genetic Algorithms are search algorithms that mimic the behaviour of natural selection to solve given problems. GAs work from a rich database of points (a population of strings) and by using mutation and recombination (crossover) operations, the population evolves toward better solutions (peaks).
procedure may climb many peaks in parallel thus; the probability of finding a false peak is reduced
with GAs as compared with the conventional methods that go from point to point like the gradient-
based methods. As the population generation proceeds, the overall population evolves toward better
solutions as individuals become adapted to the problem faced. A genetic algorithm operates on the
Darwinian principle of “survivor of the fittest”. An initial population of size \( n \) is created from a random
selection in the parameter space. As a preliminary optimization, the only element in our parameter
space is chosen to be the cascade airfoil bent angle. Each parameter set represents the individual’s
chromosomes. Each of the individuals is assigned a fitness based on how well each individual’s
chromosomes allow it to perform. At each generation (iteration) of GA’s process, fitness value
(objective function value) of every individual is evaluated and used to specify its probability of
reproduction. A new population is generated from selected parents by performing specific operators on

1. Reproduction is a process in which individual strings are copied according to their fitness
values. This implies that strings with a higher fitness value have a higher probability of
contributing in the next generation. A typical reproduction operator is the roulette-wheel
method described by Goldberg\(^8\). The reproduction process produces a mating pool.

2. Crossover proceeds in two steps. First, members in the mating pool are mated at random.
Second, each pair of strings undergoes partial exchange of their strings at a random crossing
site. This results in a pair of strings of a new generation. In this paper, a weighted average is
used:

\[
\text{Child}_1 = \text{ran}1 \times \text{Parent}_1 + (1 - \text{ran}1) \times \text{Parent}_2 \\
\text{Child}_2 = (1 - \text{ran}1) \times \text{Parent}_1 + \text{ran}1 \times \text{Parent}_2
\]  

where \( \text{Child}_1,2 \) and \( \text{Parent}_1,2 \) denote encoded design variables of the children (members of
the new population) and parents (a mated pair of the old generation), respectively. The
uniform random number \( \text{ran}_1 \) in \([0,1]\) is regenerated for every design variable. Because of Eq.
(8), the number of the initial population is assumed even.

3. Mutation is a bit change of a string that occurs during the crossover process. Mutation implies
a random walk through the string space and plays a secondary role in simple GA.
Mutation takes place at a probability of 20% (when a random number satisfies \( \text{ran}_2 < 0.1 \)).
Eqn. (8) will then be replaced by

\[
\text{Child}_1 = \text{ran}2 \times \text{Parent}_1 + (1 - \text{ran}2) \times \text{Parent}_2 + m \times (\text{ran}3 - 0.5) \\
\text{Child}_2 = (1 - \text{ran}2) \times \text{Parent}_1 + \text{ran}2 \times \text{Parent}_2 + m \times (\text{ran}3 - 0.5)
\]

where \( \text{ran}_2 \) and \( \text{ran}_3 \) are also uniform random numbers in \([0,1]\) and \( m \) determines the range of
possible mutation. In the present paper, \( m \) was set to 0.4

Fit individuals are selected for mating. Mated parents create a child with a chromosome that is some
mix of the parent’s chromosomes. The process of mating and child creation is continued until an entire
new population of size \( n \) is generated with the expectation that strong parents will create a fitter
generation. Successive generations are created until very fit individuals are obtained.

Starting from a randomly created population, there is a tendency for a few super-individuals to
dominate early on in the selection process. In this case, objective function values must be scaled back
to prevent takeover of the population by these superstrings. Therefore, the objective function values
must be scaled up to accentuate differences between population members to continue rewarding the
best performers. In the ranking method, the population is sorted according to objective function value.
The best individual receives rank 1, the second best receives 2, and so on. The fitness values are
reassigned according to rank, for example, as an inverse of their rank values. The corresponding flow
chart and the coupling of the flow solver and genetic algorithm is given in Fig.1

4. Results and Discussion

Sanz subcritical turbine cascade:

The flow solver is first validated against the test case results belonging to an incompressible
flow past a cascade. The case solution has been obtained by hodograph related methods with boundary
correction\(^12\). Fig. 2 shows the cascade grid and geometry. The analytical and numerical cascade surface
pressure coefficients are given in Fig 3. The agreement is apparent. The inlet flow angle was taken to

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be 38.54° and the outlet flow angle was determined through the Kutta condition applied at the trailing edge. The calculated flow turning is 95.15° as compared to the Sanz's exact value of 93.353°. Fig.4 shows the corresponding Mach contour for a converged 82*20 grid. The presence of the displacement thickness is clearly apparent in the wake region.

**Cascade Flow Turning Optimisation Through Genetic Algorithm**

Although, the research has now focused on multi-objective optimisation of turbine blades where the combined effect of blade thickness, pitch, stagger and flow turning angles are all taken into consideration, the preliminary study will only report the lift and lift-to-drag ratio maximization as function of a single parameter: the flow turning angle (Fig.5). The only constraint used here is the fixed airfoil thickness set at 22.8% of the chord. To see the effect of the constraint (fixed cascade thickness), the objective function has been defined by using a penalty function.

$$F = C_l \cdot \exp[-100 \times [t/c - 0.228]]$$  \hspace{1cm} (10)

The inlet flow properties are also fixed for this flow analysis. The inlet Mach number is taken as 0.343 and the relative inlet flow angle is held fixed at 38.54°. The first run of GA was made with a population size of 100 for 5 data points. The distribution of the objective functions, lift coefficient $C_l$ and Lift-to-Drag ratios are shown in Fig.6. The optimised $C_l$ and $C_l/C_d$ reached through the first 35 generations are given in Fig.7 and 8 respectively. The corresponding surface Mach number distribution is shown in Fig.9 and 10. for the optimised lift and lift-to-drag ratio respectively. The corresponding Mach numbers contours are plotted in Fig.11 and 12.

In our runs, the creep mutation and elitism has been used in the GA. This ensured that the best individual from each generation survived. Then tournament selection has been carried out. The global search procedure became more efficient as more alleles have been preserved in the first several generations.

6. Conclusion

A fast and accurate flow solver has been coupled to a robust genetic algorithm so as to optimise the lift and the lift-to-drag ratio of a subcritical turbine cascade. The main design parameters used in the optimisation algorithm are the cascade thickness, the blade pitch, the cascade stagger and flow turning angles. GA was found to be fast, accurate and very robust.

7. References


Start optimization

Random initial population
Generation = 0

Generation = Generation + 1

Generation exceeds max. generation?
YES Stop
NO
Modification of geometry:
Bending of the blade for one individual

Grid generation of the bent blade cascade

Cascade solver run & calculation of objective function for the individual

Repeat until each individual of the population is evaluated

Calculation of fitness for each individual and survival of the fittest individuals

Tournament selection and mating of most fit pairs (parents)
- binary coding
- uniform crossover
- jump mutation
- creep mutation
- niching

Repeat until entire population is replaced by children

decoding of binary codes and repeat for new generation

Fig. 1 Coupling of the genetic algorithm and flow solver
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Fig. 2 Final Euler grid result and blade geometry

Fig. 3 Analytical and numerical cascade surface mach number distributions
Fig. 4 Mach number contour of the original Sanz subcritical cascade

Fig. 5 Variation of objective functions with design parameter
Fig. 6 $C_l / C_d$ optimization

Fig. 7 $C_l$ optimization
Fig. 8 Mach number contour of the Cl / Cd optimized cascade (-5 deg. bent)

Fig. 9 Bent cascade surface mach number distributions
Fig. 10 Mach number contour of the Cl optimized cascade (+1 deg. bent)

Fig. 11 Bent cascade surface mach number distributions
Fig.12 Bending of the original Sanz subcritical cascade blade