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Analysis of disc brake squeal using the complex eigenvalue method

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Abstract

A new functionality of ABAQUS/Standard, which allows for a nonlinear analysis prior to a complex eigenvalue extraction in order to study the stability of brake systems, is used to analyse disc brake squeal. An attempt is made to investigate the effects of system parameters, such as the hydraulic pressure, the rotational velocity of the disc, the friction coefficient of the contact interactions between the pads and the disc, the stiffness of the disc, and the stiffness of the back plates of the pads, on the disc squeal. The simulation results show that significant pad bending vibration may be responsible for the disc brake squeal. The squeal can be reduced by decreasing the friction coefficient, increasing the stiffness of the disc, using damping material on the back plates of the pads, and modifying the shape of the brake pads.

Keywords: Disc brake squeal; Complex eigenvalue extraction; Friction coefficient; Stiffness; Damping ratio

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1. Introduction

Brake squeal, which usually occurs in the frequency range between 1 and 16 kHz, has been one of the most difficult concerns associated with vehicle brake systems. It causes customer dissatisfaction and increases warranty costs. Although substantial research has been conducted into predicting and eliminating brake squeal, it is still difficult to predict its occurrence due to the complexity of the mechanisms that cause brake squeal [1].

Several theories have been formulated to explain the mechanisms of brake squeal, and numerous studies have tried with varied success to apply them to the dynamics of disc brakes [2]. There are many models for analysing disc brake squeal. For example, the effect of surface topography of the pad/disc assembly on squeal generation was reported [3] and a distributed-parameter model of a disc brake has been developed to simulate friction-induced vibrations in the form of high-frequency squeal [4]. A two-degree-of-freedom model has been used to investigate the basic mechanisms of instability of the disc brake system and demonstrates the conditions necessary for preventing the instability [5]. Brake squeal has also been studied from an energy perspective using feed-in energy analysis and results indicate a squeal tendency of the brake system [6]. The use of viscoelastic material (damping material) on the back of the back plates of the pads can be effective in reducing squeal when there is significant pad bending vibration [7] and another reported effective method is to modify the shape of the brake pads to change the coupling between the pads and the disc [8].

Brake noise is mainly caused by friction-induced dynamic instability. There are two main categories of numerical methods that are used to study this problem: (1) transient dynamic analysis and (2) complex eigenvalue analysis. Currently the complex eigenvalue method is preferred and widely used in predicting the squeal propensity of the brake system including damping and contact [9–12], since the transient dynamic analysis is computationally expensive. The main idea of the complex eigenvalue method involves symmetry arguments of the stiffness matrix and the formulation of a friction coupling. This method is more efficient and provides more insight to the friction-induced dynamic instability of the disc brake system.

In the present study, an investigation of disc brake squeal is performed by using the new complex eigenvalue capability of the finite element (FE) software ABAQUS version 6.4 [13]. This FE method uses nonlinear static analysis to calculate the friction coupling prior to the complex eigenvalue extraction, as opposed to the direct matrix input approach that requires the user to tailor the friction coupling to stiffness matrix, Thus, the effect of non-uniform contact and other nonlinear effects are incorporated. A systematic analysis is done to investigate the effects of system parameters, such as the hydraulic pressure, the rotational velocity of the disc, the friction coefficient of the contact interactions between the pads and the disc, the stiffness of the disc, and the stiffness of the back plates of the pads, on the disc squeal. The simulations performed in this work present a guideline to reduce the squeal noise of the disc brake system.

2. Methodology and numerical model

2.1. Complex eigenvalue extraction

For brake squeal analysis, the most important source of nonlinearity is the frictional sliding contact between the disc and the pads. ABAQUS allows for a convenient, but general definition of contact interfaces by specifying the contact surface and the properties of the interfaces. ABAQUS version 6.4 has developed a new approach of complex eigenvalue analysis to simulate the disc brake squeal. Starting from preloading the brake, rotating the disc, and then extracting natural frequencies and complex eigenvalues, this new approach combines all steps in one seamless run [13]. The complex eigenproblem is solved using the subspace projection method, thus a natural frequency extraction must be performed first in order to determine the projection subspace. The governing equation of the system is

$$\boldsymbol{M}\ddot{\boldsymbol{x}} + \boldsymbol{C}\dot{\boldsymbol{x}} + \boldsymbol{K}\boldsymbol{x} = 0, \tag{1}$$

where $\boldsymbol{M}$ is the mass matrix, $\boldsymbol{C}$ is the damping matrix, which includes friction-induced contributions, and $\boldsymbol{K}$ is the stiffness matrix, which is unsymmetric due to friction. The governing equation can be rewritten as

$$(\mu^2\boldsymbol{M} + \mu\boldsymbol{C} + \boldsymbol{K})\boldsymbol{\Phi} = 0, \tag{2}$$

where μ is the eigenvalue and Φ is the corresponding eigenvector. Both eigenvalues and eigenvectors may be complex. In order to solve the complex eigenproblem, this system is symmetrized by ignoring the damping matrix $\boldsymbol{C}$ and the unsymmetric contributions to the stiffness matrix $\boldsymbol{K}$. Then this symmetric eigenvalue problem is solved to find the projection subspace. The N eigenvectors obtained from the symmetric eigenvalue problem are expressed in a matrix as $[\phi_1, \ldots, \phi_N]$. Next, the original matrices are projected onto the subspace of N eigenvectors

$$\boldsymbol{M}^* = [\phi_1, \ldots, \phi_N]^{\mathrm{T}}\boldsymbol{M}[\phi_1, \ldots, \phi_N], \tag{3a}$$

$$\boldsymbol{C}^* = [\phi_1, \ldots, \phi_N]^{\mathrm{T}}\boldsymbol{C}[\phi_1, \ldots, \phi_N] \tag{3b}$$

and

$$\boldsymbol{K}^* = [\phi_1, \ldots, \phi_N]^{\mathrm{T}}\boldsymbol{K}[\phi_1, \ldots, \phi_N]. \tag{3c}$$

Then the projected complex eigenproblem becomes

$$(\mu^2\boldsymbol{M}^* + \mu\boldsymbol{C}^* + \boldsymbol{K}^*)\boldsymbol{\Phi}^* = 0. \tag{4}$$

Finally, the complex eigenvectors of the original system can be obtained by

$$\boldsymbol{\Phi} = [\phi_1, \ldots, \phi_N]^{\mathrm{T}}\boldsymbol{\Phi}^*. \tag{5}$$

A more detailed description of the algorithm may be found in [13]. The complex eigenvalue μ, can be expressed as $\mu = \alpha \pm \mathrm{i}\omega$ where α is the real part of μ, $\mathrm{Re}(\mu)$, indicating the stability of the system, and ω is the imaginary part of μ, $\mathrm{Im}(\mu)$, indicating the mode frequency. The generalized displacement of the disc system, x, can then be expressed as

$$x = A\,\mathrm{e}^{\mu t} = \mathrm{e}^{\alpha t}(A_1 \cos \omega t + A_2 \sin \omega t). \tag{6}$$

This analysis determines the stability of the system. When the system is unstable, α becomes positive and squeal noise occurs. An extra term, damping ratio, is defined as $-\alpha/(\pi|\omega|)$. If the damping ratio is negative, the system becomes unstable, and vice versa. The main aim of this analysis is to reduce the damping ratio of the dominant unstable modes.

2.2. Finite element model

A disc brake system consists of a disc that rotates about the axis of a wheel, a calliper–piston assembly where the piston slides inside the calliper, that is mounted to the vehicle

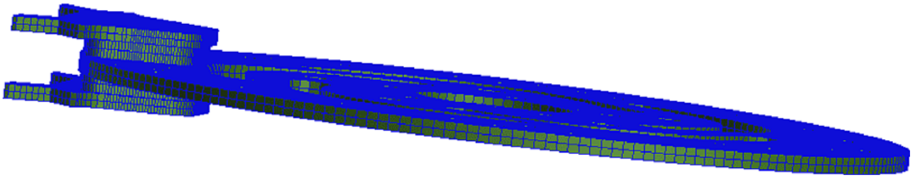

Fig. 1. Geometry and finite element mesh of the simplified disc brake system.

suspension system, and a pair of brake pads. When hydraulic pressure is applied, the piston is pushed forward to press the inner pad against the disc and simultaneously the outer pad is pressed by the calliper against the disc. The brake model used in this study is a simplified version of a disc brake system which consists of a disc and a pair of brake pads. The disc has a diameter of 292 mm and a thickness with typical value of 5.08 mm and is made of cast iron. The pair of brake pads, which consist of contact plates and back plates, are pressed against the disc in order to generate a friction torque to slow the disc rotation.

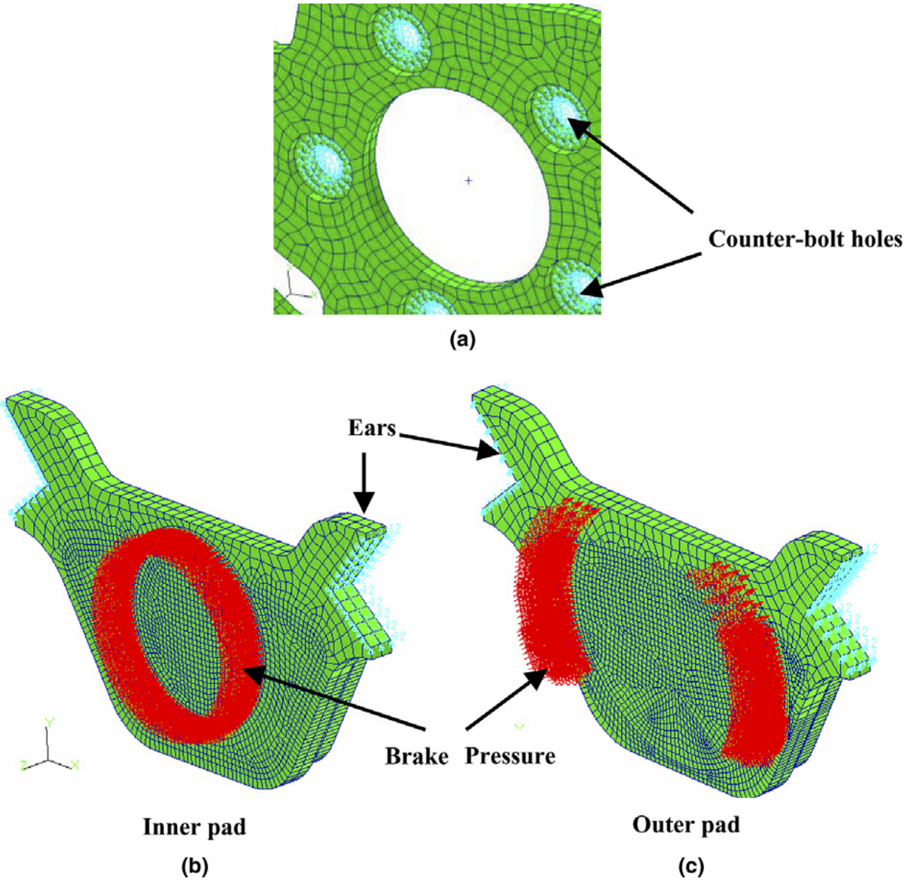

Fig. 2. Constraints and loadings of the disc brake system.

The contact plates are made of an organic friction material and the back plates are made of steel. The FE mesh is generated using three-dimensional continuum elements for the disc and pads as shown in Fig. 1, where a fine mesh is used in the contact regions. The friction contact interactions are defined between both sides of the disc and the contact plates of the pads. A constant friction coefficient and a constant angular velocity of the disc are used for simulation purposes. Figs. 2(a)–(c) present the constraints and loadings for the pads and disc assembly. The disc is completely fixed at the five counter-bolt holes as shown in Fig. 2(a) and the ears of the pads are constrained to allow only axial directional movements as shown in Figs. 2(b) and (c). The calliper–piston assembly is not defined in the simplified model of the disc brake system, hence the hydraulic pressure, which has a typical value of 0.5 MPa, is directly applied to the back plates at the contact regions between the inner pad and the piston and between the outer pad and the calliper as shown in Figs. 2(b) and (c), and it is assumed that an equal magnitude of force acts on each pad. The analysis procedure contains the following four steps: (1) nonlinear static analysis for the application of brake pressure; (2) nonlinear static analysis to impose a rotational velocity on the disc; (3) normal mode analysis to extract the natural frequency to find the projection subspace; and (4) complex eigenvalue analysis to incorporate the effect of friction coupling.

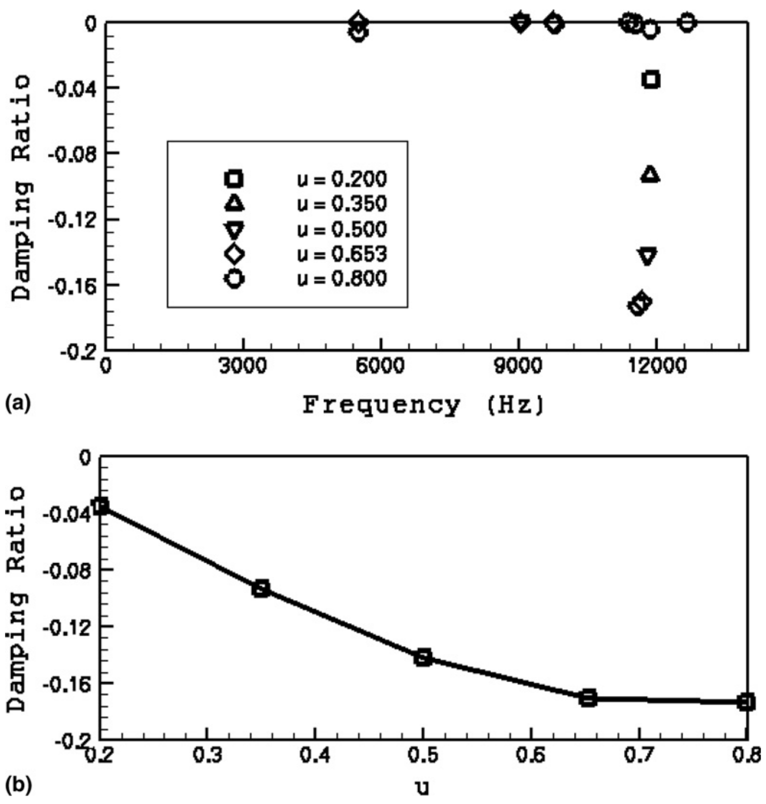

Fig. 3. (a) Variation of the damping ratio with frequency for different friction coefficients; (b) variation of the damping ratio with friction coefficient at frequency 12 kHz.

3. Results and discussion

The effects of the system parameters, such as the hydraulic pressure P, the rotational velocity of the disc W, the friction coefficient of the contact interactions between the pads and the disc u, the stiffness of the disc, and the stiffness of the back plates of the pads, on the disc squeal are investigated by the simulation model. The effect of the stiffness of the disc can be changed by varying Young's modulus E_D and the disc thickness T_D of the disc while the effect of the stiffness of the back plates of the pads can be changed by varying Young's modulus E_P of the back plates of the pads. The complex eigenvalue analysis is performed up to 13 kHz which is the range of squeal occurrence for the present disc model. As mentioned previously, if the damping ratio is negative, the system becomes unstable, and vice versa; when the disc system is unstable, the squeal propensity increases with an increased value of the damping ratio (absolute values are used). For clarity, only negative values of the damping ratio are plotted. The typical values for the system parameters used in the simulation are: $P = 0.5$ MPa, $W = 1.5$ rad/s, $u = 0.653$, $E_D = 219.669$ GPa, $T_D = 5.08$ mm, and $E_P = 210$ GPa. Analysis is carried out by changing the values of each parameter while retaining the respective typical values for the others.

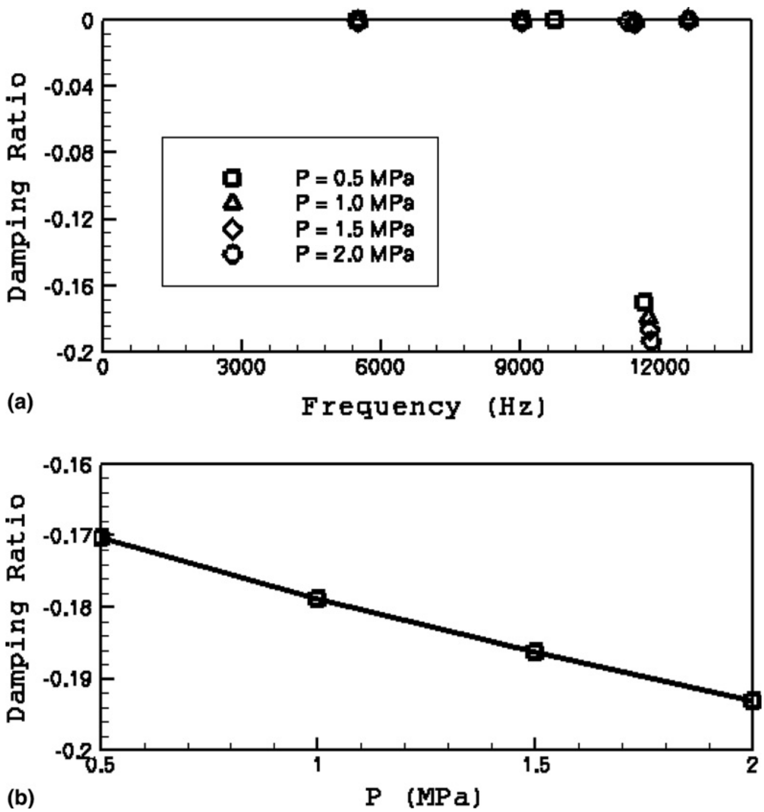

Fig. 4. (a) Variation of the damping ratio with frequency for different hydraulic pressures; (b) variation of the damping ratio with hydraulic pressure at frequency 12 kHz.

3.1. Effect of friction coefficient

Disc squeal is believed to be caused mainly by friction-induced dynamic instability. This section presents the effect of the friction coefficient of the contact interactions between the pads and the disc on the disc squeal, in which the friction coefficient u varies from 0.2 to 0.8. Fig. 3(a) shows results in the form of the damping ratio as a function of frequency for different friction coefficients. It can be seen that the major squeal frequency is approximately 12 kHz. The value of the damping ratio is decreased significantly with a decrease of the friction coefficient as shown in Fig. 3(b) at a frequency of 12 kHz. It is understandable that with an increase in the friction coefficient, there is an accompanying increase in the instability of the system, thus an increase in the damping ratios. This means that the most fundamental method of eliminating brake squeal is to reduce the friction between the pads and the disc. However, this obviously reduces braking performance and is not a preferable method to employ.

3.2. Effect of hydraulic pressure

The effect of the hydraulic pressure P on the squeal propensity is studied by varying P from 0.5 MPa to 2.0 MPa. Fig. 4(a) shows the change of the damping ratio with frequency

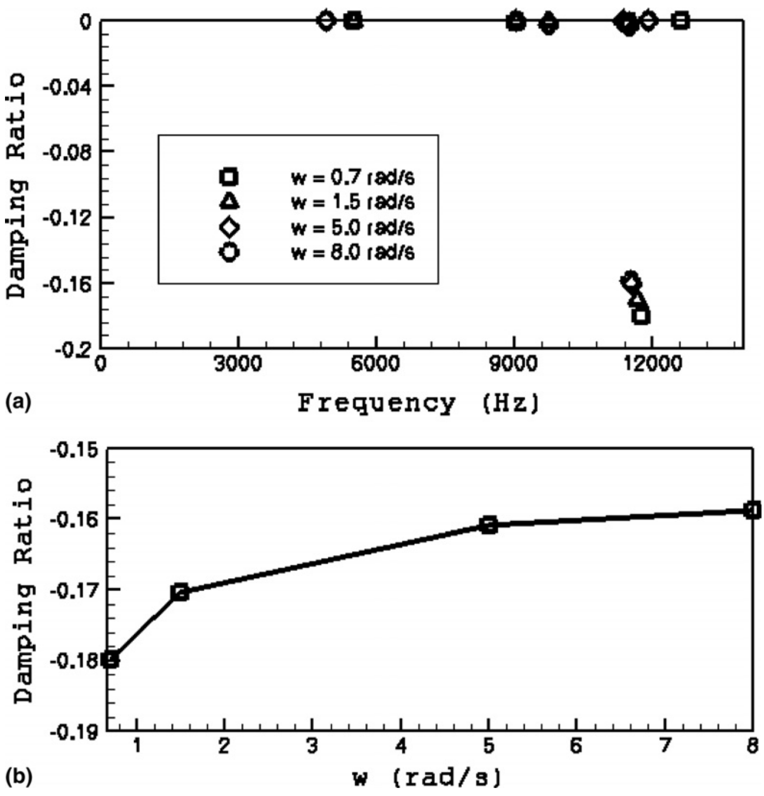

Fig. 5. (a) Variation of the damping ratio with frequency for different rotational velocities of the disc; (b) variation of the damping ratio with rotational velocity of the disc at frequency 12 kHz.

for different hydraulic pressures. The major squeal frequency is approximately 12 kHz. It can be seen from Fig. 4(b) that with an increase in P, the value of the damping ratio is increased, so the squeal propensity is increased. This is due to a larger hydraulic pressure inducing more friction between the pads and the disc. However, the simulation results also show that the effect of the hydraulic pressure on the disc brake squeal is not significant because the value of the damping ratio only changes from 0.17 to 0.193 when P increases from 0.5 MPa to 2.0 MPa.

3.3. Effect of rotational velocity of the disc

Fig. 5(a) presents the variation of the damping ratio with the frequency for different disc angular velocities W (0.7–8.0 rad/s). The dominant squeal frequency is approximately 12 kHz. As the angular velocity increases, the value of the damping ratio gradually decreases. However, as with the previous case, when changing the hydraulic pressure, the effect of changing the angular velocity on the squeal propensity is also not obvious: this can be seen from Fig. 5(b) which shows the value of the damping ratio varies with an increase in the rotational velocity of the disc.

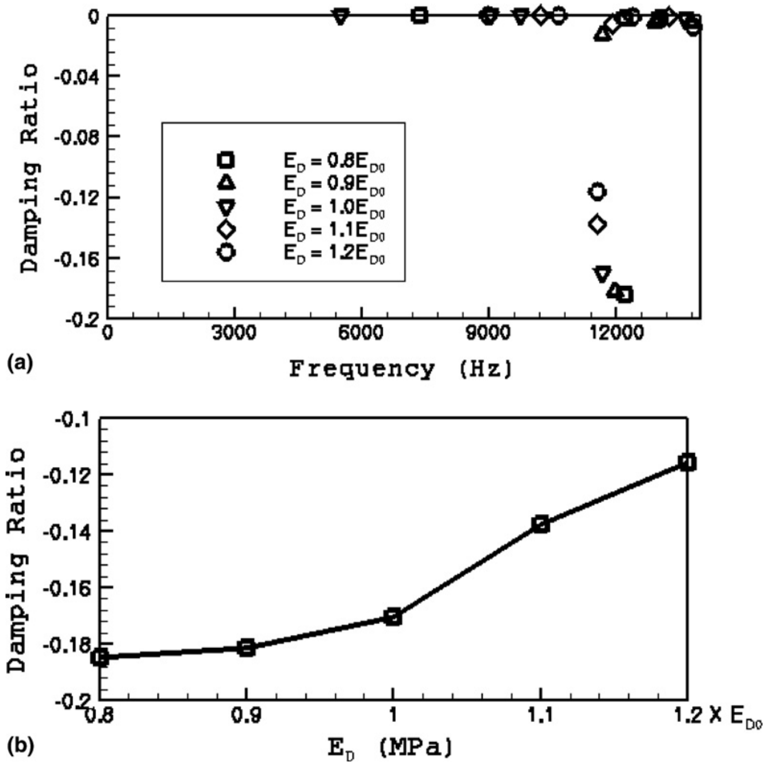

Fig. 6. (a) Variation of the damping ratio with frequency for different Young's moduli of the disc; (b) variation of the damping ratio with Young's moduli of the disc at frequency 12 kHz.

3.4. Effect of stiffness of the disc

The effect of the stiffness of the disc on the disc brake squeal is studied by changing Young's modulus E_D and the thickness T_D of the disc. Fig. 6(a) shows results of the damping ratio versus frequency for different Young's modulus E_D, i.e. $E_D = 0.8E_{D0}$, $0.9E_{D0}$, $1.0E_{D0}$, $1.1E_{D0}$ and $1.2E_{D0}$, where E_{D0} is the typical value of Young's modulus of the disc, which is 219.669 GPa. It can be seen that the major squeal frequency does not change for different disc Young's moduli. The value of the major squeal frequency is approximately 12 kHz. As Young's modulus E_D is increased and hence as the stiffness of the disc is increased, the value of the damping ratio decreases greatly. Fig. 6(b) presents the damping ratio versus Young's modulus of the disc at a frequency of 12 kHz. It is found that a larger disc stiffness can reduce the squeal propensity of the disc system. It is believed that a stiffening of the disc can reduce the disc vibration magnitude, as a result, the squeal propensity of the disc system can be reduced. The stiffness of the disc is also changed by varying its thickness T_D. Four cases were studied, i.e. $T_D = 0.9T_{D0}$, $1.0T_{D0}$, $1.1T_{D0}$ and $1.2T_{D0}$, where $T_{D0} = 5.08$ mm is the typical value for disc thickness. Fig. 7(a) shows results of the damping ratio plotted against frequency for different disc thicknesses and Fig. 7(b) presents the damping ratio versus disc thickness at a frequency of 12 kHz. The thicker the

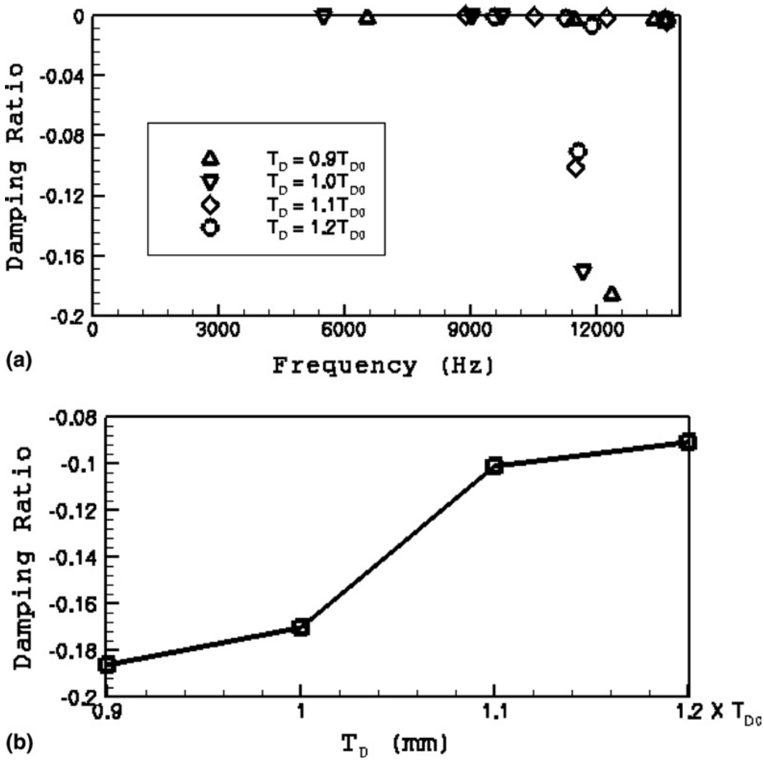

Fig. 7. (a) Variation of the damping ratio with frequency for different disc thicknesses; (b) variation of the damping ratio with thickness of the disc at frequency 12 kHz.

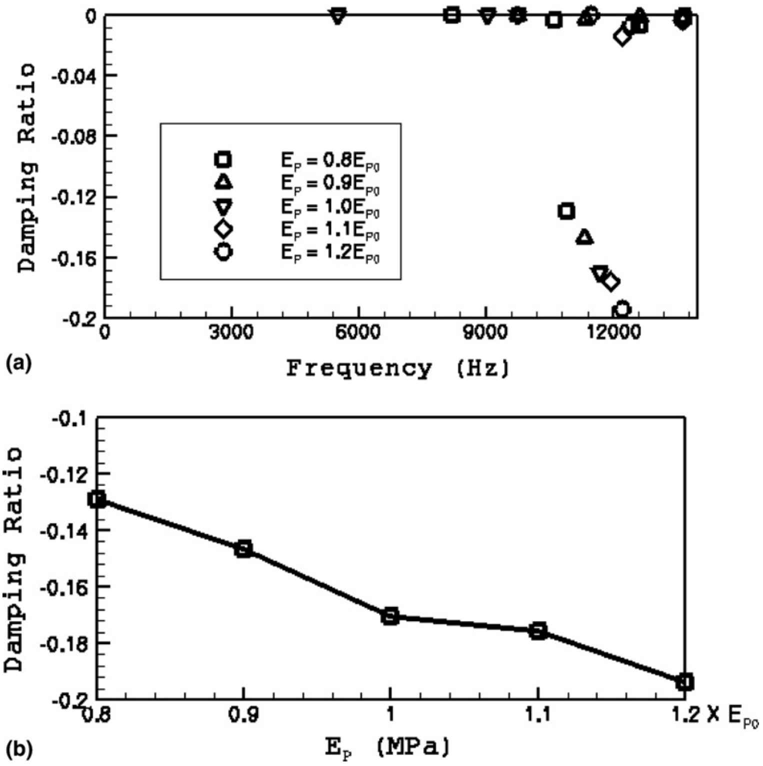

Fig. 8. (a) Variation of the damping ratio with frequency for different Young's moduli of the back plates of the pads; (b) variation of the damping ratio with Young's moduli of the back plates of the pads at frequency 12 kHz.

disc, the higher its stiffness, the smaller the damping ratio, and thus the lower the squeal propensity.

3.5. Effect of stiffness of the back plates of the pad

Brake pads consist of contact plates which are made of a friction material and back plates. In this study, the effect of Young's modulus E_P of the back plates of the pads on the disc squeal is investigated, in which $E_P = 0.8E_{P0}$, $0.9E_{P0}$, $1.0E_{P0}$, $1.1E_{P0}$ and $1.2E_{P0}$, where $E_{P0} = 210$ GPa, is the typical value of Young's modulus for the back plates of pads. Fig. 8(a) shows results of the damping ratio versus frequency for different Young's moduli E_P. It can be seen that the dominant squeal occurs at a frequency of approximately 12 kHz. As Young's modulus E_P, is increased, corresponding to an increase in stiffness of the back plates of the pads, the value of the damping ratio increases significantly as shown in Fig. 8(b); here the variation of the damping ratio with Young's modulus of the back plates at a frequency of 12 kHz is shown. This important observation implies that the stiffer back plates of pads cause a higher squeal propensity. This is so since the friction material connected to the back plates is very soft compared with the back plate material. Hence the higher the stiffness of the back plates, the greater the uneven deformation and vibration magnitude of the pad, and hence the higher the damping ratio. So an effective

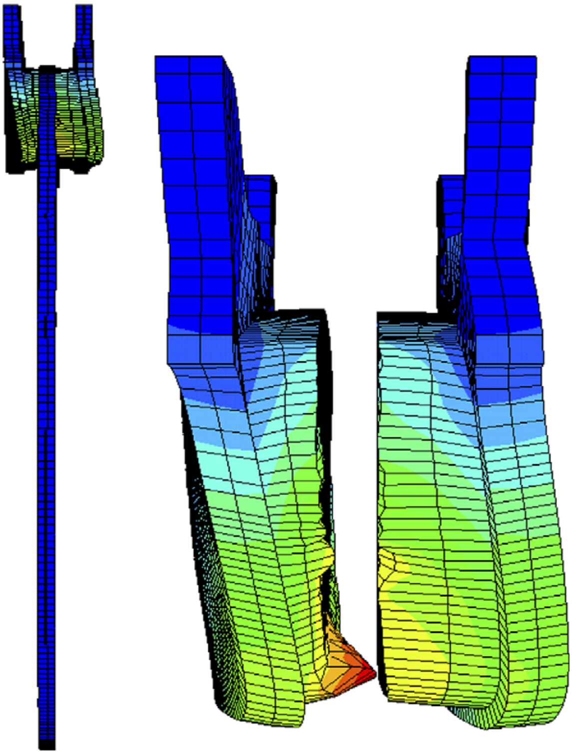

Fig. 9. Vibration mode of the disc brake system at frequency 12 kHz.

method to reduce squeal propensity of disc brake system is to use a damping material for the back plates of the pads.

3.6. Unstable modes of disc brake system

The simulation results show that for all the cases owe large damping ratios, the unstable frequencies are approximately 12 kHz. There is a significant pad bending vibration for these cases. Fig. 9 gives an example of the vibration mode of the disc brake system at a frequency of 12 kHz, where all the system parameters are the typical values. It can be seen that the disc has only slight out-of-plane modes of vibration as shown in Fig. 9(a), but the pads have serious out-of-plane modes of vibration which occur mainly at the bottom parts of the pads as shown in Fig. 9(b). This suggests that the brake pads may be the source of the disc brake squeal. So methods which can reduce the pad bending vibration should be used to eliminate the disc squeal. One of the methods reported is to use viscoelastic material (damping material) on the back of the back plates of the pads [7] and another effective method is to modify the shape of the brake pads to change the coupling between the pads and the disc [8]. Except the unstable vibration modes which occur at frequency 12 kHz and are caused mainly by the pads vibration, the other unstable vibration modes are caused mainly by the disc vibration. Figs. 10(a) and (b) give an example of the unstable vibration

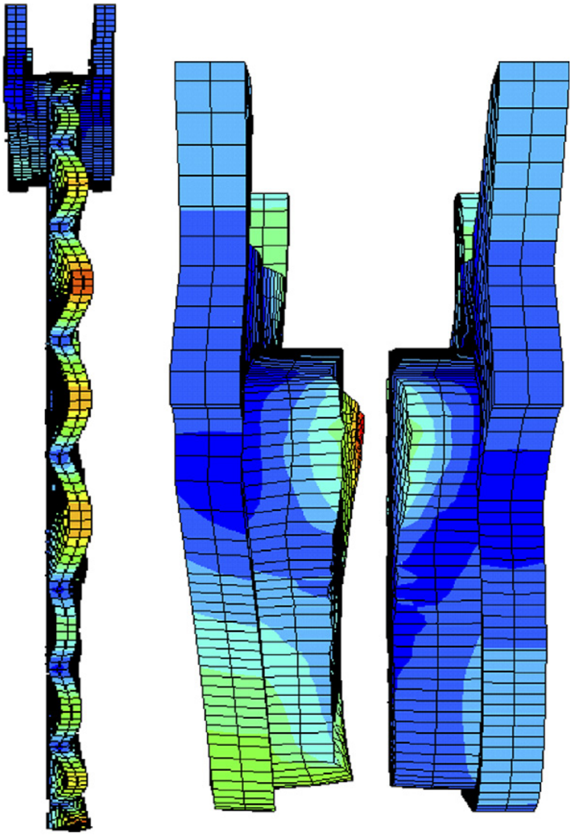

Fig. 10. Vibration mode of the disc brake system at frequency 9766 Hz.

mode of the disc brake system at the frequency of 9766 Hz, where all the system parameters are the typical values. It can be seen that the disc has significant out-of-plane vibration compared with the vibration of pads.

4. Conclusion

Friction-induced disc brake squeal is investigated using the new function of ABAQUS version 6.4, which combines a nonlinear static analysis and a complex eigenvalue extraction method. The nonlinear effects can be taken into account in the preloading steps in order to more accurately model a deformed configuration at which a complex eigenvalue analysis is performed. The systematic analysis here shows that significant pad bending vibration may be responsible for causing the disc brake squeal and the major squeal frequency is approximately 12 kHz for the present disc brake system. The effects of the friction between the pads and the disc, the stiffness of the disc, and the stiffness of the back plates of the pads, on disc squeal, are significant, but the effects of the hydraulic pressure and the angular velocity of the disc on disc squeal are not obvious. The squeal can be reduced by decreasing the friction coefficient, increasing the stiffness of the disc, using damping material on the back of the pads, and modifying the shape of the brake pads.

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