Large Eddy Simulation for Turbulent Mixing in Elliptic Jets with Round Center-body

Mihai Mihăescu*, David Munday† and Ephraim Gutmark‡
University of Cincinnati, 745 Baldwin Hall, ML 0070, P.O. Box 210070, Cincinnati, OH, 45221, USA

Separate-flow exhaust nozzle systems generate jets from round nozzles that include a conical plug. The present research shows results of unsteady simulations of turbulent hot jets issued from elliptic nozzles with a conic center-body. Three geometrical configurations of the elliptic nozzle are investigated keeping the same conic plug. All designs considered were intended to match the jet stream exit area, mass flow, and thrust of an existing round conical nozzle. The first configuration in the study (C1@3:1AR) was designed with the expectation of producing a jet which would take on a 3:1 aspect ratio (AR) elliptic cross-section downstream of the center-body. For a round conical plug nozzle the inner surface of the nozzle towards exit is axisymmetric with reference to the nozzle center-line and conical. Naturally, the elliptic plug nozzle is not axisymmetric. In the major axis plane the inner surface of the elliptic nozzle towards the exit has a slope close to zero, while in the minor axis plane the slope is steeper that it would be for a corresponding round nozzle. This forces the flow stream towards the conic plug in the minor axis plane. It was observed that the 3:1AR elliptic plug nozzle (C1@3:1AR) generates a bifurcated jet. The second and the third elliptic plug nozzle geometries (C2@3:1ARnf and C3@2:1ARnf) were intended to find how the jet behavior is influenced by not forcing the flow towards the conic plug in the minor axis plane and by changing the aspect ratio of the elliptic plug nozzle to 2:1. Large Eddy Simulation (LES) approach was used for the turbulence flow modeling. In the major axis plane the largest jet spreading was found for the 3:1AR elliptic plug nozzle (C1@3:1AR), while in the minor axis plane the jet exhausting from the 2:1AR elliptic plug nozzle (C3@2:1ARnf) spread the most. The numerical results obtained for the 3:1 AR elliptic jet configuration (C1@3:1AR) were compared with the experimental data obtained for the same setup using Particle Image Velocimetry (PIV). A good agreement was found between the LES results and the experimental measurements.

I. Introduction

Turbulent jets are part of essential scientific and engineering problems with applications in mixing, heat transfer, combustion, propulsion, and aero-acoustical flows. A large number of experimental and numerical investigations have been carried out to study the various characteristics of jet flows. Round jets have been well documented and based on the experimental data obtained using stationary hot-wire, flying hot-wire or Laser-Doppler Anemometry (LDA) techniques, analytical expressions for the center-line decay in the self-preserving region of the jet were derived.1–4

As compared with the jets exhausting from axisymmetric nozzles, the entrainment and spreading characteristics of non-circular jets are considerably different.5–10 These experimental studies show that as the jet develops and spreads, its cross-section can evolve through shapes similar to that at the nozzle exit but with its axis switched. It was revealed that the downstream evolution of the elliptic jets is governed by the flow characteristics at the exhaust port. Specific conditions that must exist at the near-field region of a non-circular jet were formulated to enhance vortex dynamics that lead to self-induction and eventually to axis-switching.11 It is believed that this mechanism is the main cause for the observed enhanced mixing

*Research Associate, PhD, Dept. of Aerospace Engineering and Engineering Mechanics, Cincinnati OH, AIAA Member.
†Graduate Student, Dept. of Aerospace Engineering and Engineering Mechanics, Cincinnati OH, AIAA Member.
‡Professor, PhD, Dept. of Aerospace Engineering and Engineering Mechanics, Cincinnati OH, AIAA Associate Fellow.
with rectangular and elliptic jets relative to comparable circular jets. The mass entrainment ratio associated with a non-circular jet may be several times higher as compared to a circular jet as shown by Ho and Gutmark. Under other conditions high aspect ratios elliptic jets have been found to bifurcate after an initial axis-switching as shown by Hussain and Hussain.

Computational Fluid Dynamics studies of compressible and/or incompressible round and/or non-circular turbulent jets, have been performed using Reynolds Averaged Navier-Stokes (RANS/unsteady RANS), Direct Numerical Simulation (DNS), Large Eddy Simulation (LES), or Detached Eddy Simulation (DES). Among these numerical techniques it was proved that LES represents the most realistic approach since is less expensive than DNS and in the same time is able to capture the fluid flow dynamics. LES was successfully used to identify the vortex ring bifurcation phenomena and to demonstrate vorticity geometries characterizing the near-field of different aspect ratio rectangular jets.

Most of the published studies on non-circular jets refer to nozzles without conical plug or center-body within their exit planes. Several important engineering problems like the separate-flow exhaust systems from jet engines generate jets from nozzles that must include center-bodies. The present numerical study represents a research effort into how the introduction of the center-body changes the behavior of elliptic jets. Using the LES approach, first an isothermal 2:1AR elliptic jet without center-body is investigated, the LES data being contrasted against experimental measurements. Secondly, the flows associated with three elliptic plug nozzle configurations (i.e. C1@3:1AR, C2@3:1ARnf, and C3@2:1ARnf) are analyzed in order to identify how the geometrical features and the aspect ratio of the elliptic nozzle affect the jet development downstream. Validation against experimental data obtained by Munday and Gutmark are performed for the 3:1 aspect ratio elliptic plug nozzle (C1@3:1AR).

II. Governing Equations and Numerical Algorithm

LES is used to handle the mixing of the turbulent hot jet exhausting the elliptic nozzle system with the surrounding stagnant cold air. The equations used to solve the problem are the non-dimensional continuity, momentum, and energy, the last one written in terms of temperature ($T$). These equations are written in a Cartesian coordinate system using index notation and the Einstein summation convention:

$$\frac{\partial p}{\partial t} + \frac{\partial \rho u_i}{\partial x_i} = 0$$  \hspace{1cm} (1)

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{1}{Re} \frac{\partial^2 u_i}{\partial x_i \partial x_j}$$  \hspace{1cm} (2)

$$\frac{\partial T}{\partial t} + u_i \frac{\partial T}{\partial x_i} = \frac{1}{Pr \cdot Re} \frac{\partial^2 T}{\partial x_i \partial x_i}$$  \hspace{1cm} (3)

where $Re$ and $Pr$ denote the Reynolds and Prandtl numbers, respectively. The normalized temperature used in the computations is given by:

$$T = \frac{\tilde{T} - T_{ref}}{T_{max} - T_{ref}}$$  \hspace{1cm} (4)

where $\tilde{T}$ is the instantaneous, local temperature, $T_{max}$ is the maximum temperature and $T_{ref}$ is the reference temperature.

In order to close the system of equations a second order polynomial dependence between density and temperature is employed instead of the classical state equation:

$$\rho = A_1 + A_2 \cdot T + A_3 \cdot T^2$$  \hspace{1cm} (5)

In Eqn. (5), $A_1$, $A_2$, and $A_3$ are the coefficients of the polynomial function which can be adjusted, depending on the temperature range.

For the spatial discretization of the equations finite-differences are used on a Cartesian, staggered grid. Local mesh refinements are implemented to enhance the grid resolution in the regions with large gradients in the flow variables and to keep the computational effort in reasonable limits. The blocked-cell approach is used to handle the geometry of the nozzle. Turbulence is modeled by LES, the sub-grid scale (SGS) terms being not modeled explicitly. When high spatial resolution is used, one may omit the explicit SGS term.
The dissipation of the numerical scheme accounts for the SGS dissipation. A third order scheme by Rai and Moin\textsuperscript{24} is used to discretize the convective terms in the momentum equations. Fourth order schemes are employed to discretize the diffusive terms. The leading term in the truncation error comes from the third order scheme and it is expressed as:

\[
\tau_{ij,j} = -\frac{1}{12} \bar{\rho} \bar{u}_j h^3 \frac{\partial^4 \bar{u}_i}{\partial x_j^4} \tag{6}
\]

where \(h\) represents the size of the computational cell. The truncation error contains the fourth derivative of the velocity that physically may be interpreted as high frequency dissipation or diffusion, as was shown by Kawamura and Kuwahara.\textsuperscript{25} This term also allows “back scatter” (the effect of the small scales on the large ones).\textsuperscript{17} The approach has been successfully applied among others by Kawamura and Kuwahara,\textsuperscript{25} Olsson and Fuchs,\textsuperscript{17} and Gullbrand et al.\textsuperscript{26} Comparisons between the results obtained with and without explicit (different) SGS terms showed that, for sufficiently fine grid resolution, the SGS terms might be neglected.\textsuperscript{27} The integration in time of the momentum equations is performed through an implicit scheme. An iterative pressure-correction method with simultaneous velocity and pressure update is used. The temporal discretization of the energy equation is performed with an explicit three-step, low-storage Runge-Kutta scheme. Convergence, in each time step, is enhanced by using the multi-grid technique.

### III. Cases Formulation

First, an isothermal low Mach number turbulent jet ejected in a large rectangular domain from an elliptic nozzle with a 2:1AR and without center-body is simulated by using the previously described LES in-house solver. This benchmark case for which extensive experimental measurements exist\textsuperscript{7} was used to verify the LES abilities in predicting the axis-switching phenomenon that was experimentally proved to occur by Ho and Gutmark.\textsuperscript{7} In this particular case the Reynolds number \(Re = W_o \cdot D_{eq} / \nu\) based on the jet exit velocity \(W_o\), the equivalent diameter \(D_{eq}\), and the kinematic viscosity of the air \(\nu\) was \(4 \times 10^4\). The equivalent diameter for the elliptical nozzle \(D_{eq} = 2(a \cdot b)^{1/2}\) corresponds to that of a circle with an equivalent area (for a circle \(a = b = \text{radius}\)), where the major and minor diameters of the ellipse are \(2a\) and \(2b\), respectively.
The computational domain for this set-up had a square cross-section of 7.5 by 7.5 equivalent diameters and a length of 19 equivalent diameters.

After showing that the LES solver predicted the axis-switching for the 2:1 AR elliptic nozzle without center-body, in the second part of the paper three elliptic plug nozzle configurations are investigated keeping the same center-body and modifying only the geometry of the elliptic nozzle. All designs considered were intended to match the jet stream exit area, mass flow, and thrust of an existing round conical nozzle. However, for a round conical plug nozzle the inner surface of the nozzle towards exit is axisymmetric with reference to the nozzle center-line and conical in order to have the flow exiting tangent to the conic surface of the center-body. Naturally, the elliptic plug nozzle is not axisymmetric. It is characterized by the fact that in the minor axis plane the hot jet stream is forced by the inner shape of the nozzle (which is steeper that it would be for a corresponding round nozzle) towards the conical plug at a steep angle (i.e. at the exit the flow is not tangent to the conical plug). In the major axis plane the inner surface of the elliptic nozzle towards the exit has a slope close to zero. The concept design of the elliptic plug nozzle is illustrated in Figures 1a and 1b, which represent sketches in the major and minor axis planes of the plug nozzle.

The first elliptic plug nozzle configuration considered in the study, identified as C1@3:1AR, was designed so that the ratio of the exit heights $h_1$ to $h_2$ (see Figures 1a and 1b) would be 3:1 with the expectation of producing a jet which would take on a 3:1 aspect ratio elliptic cross-section downstream of the center-body. The conic plug, common for all cases, has a half-angle of 16 degrees and extends 1.79 equivalent jet diameters ($D_{eq}$) beyond the exit plane of the nozzle. The jet equivalent diameter was based on the jet stream exit area.

The second (3:1AR) and the third (2:1AR) elliptic plug nozzle configurations were intended to find how the jet behavior is influenced by not forcing in the minor axis plane the flow stream towards the conic plug. For these cases (C2@3:1ARnf and C3@2:1ARnf) the inner shape of the elliptic nozzle was modified so that in the minor axis plane the exhausting flow stream is not impinging anymore at a steep angle the surface of the conic plug, allowing the flow to follow unhampered the conical shape of the center-body.

The considered computational domains for the elliptic plug nozzle set-ups (roughly $7 \times 10^6$ computational cells, by using local mesh refinements) have a square cross section of roughly $15D_{eq} \times 15D_{eq}$ and a length of $30D_{eq}$. Details of the computational Cartesian grid around the 3:1 AR elliptic plug nozzle geometry (C1@3:1AR) are depicted in Figure 2. As boundary conditions at the inlet port, constant values for the flow velocity and temperature were specified to match the desired conditions, i.e. Nozzle Pressure Ratio of 1.8 and the jet temperature of 378K. No-slip boundary conditions for the velocity and constant value for temperature are set at the solid surfaces and the flux conserving zero gradient boundary condition is applied at the outlet of the computational domain. For all cases the LES results were statistically averaged for a period of 30000 time-steps, achieving a converged solution at each time-step.

IV. Results

The time-averaged velocity magnitude distribution presented in Figure 3 for the 2:1AR elliptic nozzle without center-body shows that the phenomenon of axis-switching was captured by the LES solver. The velocity data is shown along cross-sectional planes located at seven different downstream positions from the nozzle exit (2, 4, 6, 8, 10, and 12 equivalent diameters). The cross-section of the jet changes its shape as the jet develops downstream in the stream-wise $z$ direction. Close to the inlet, the jet keeps its elliptical shape with the same orientation as the nozzle, but the axis switching is obvious at 12 equivalent diameters stream-wise distance from the nozzle. Quantitative comparisons between the LES data and Ho and Gutmark’s measurements for this case are depicted in Figures 4, 5, and 6.

The normalized mean axial velocity ($W/W_o$) and its fluctuating component ($W_{rms}/W_o$) along the normalized center line of the elliptic jet ($z/a$) as predicted by the LES and as measured are compared in Figures 4a and 4b. A fair agreement is found between the LES results and the measurements along the jet center line. The cross-sectional evolution of the 2:1 AR elliptic jet without center-body in the major ($x/a$) and minor ($y/a$) axis planes is illustrated in Figures 5 and 6. The time-averaged axial velocity profiles in the major and minor axis planes normalized by the jet exit velocity are presented in Figures 5a and 5b. The data are extracted along lines located at five stream-wise positions ($z = 0.5a, 2a, 4a, 10a, and 12a$) from the inlet plane. The way in which the 2:1AR elliptic jet without center-body spreads is depicted in Figure 6 by plotting the jet width in the major and minor planes at the time-averaged axial velocity value ($W$) that is half of the mean center-line velocity value ($W_c$), $W/W_c = 0.5$. While in the experimental data the axis switching occurred at about $3.5D_{eq}$ downstream distance from the nozzle exit ($z \sim 5a$), in the LES
predictions the axis switching occurred at roughly $7D_{eq}$. The discrepancies seen between the two sets of data may be explained by not matching in the simulations the exact velocity profile developed at the nozzle exit in the experiments and by the confinement effects even if the rectangular computational domain used in the calculations was considered large enough.

This successful validation of the LES solver provided support for investigating the elliptic nozzle configurations with center-body (C1@3:1AR, C2@3:1ARnf, and C3@2:1ARnf).

The overall flow behavior of the jet exhausting an elliptic plug nozzle system is presented by showing detailed LES data obtained for the C1@3:1AR case for which experimental PIV measurements were performed.\textsuperscript{22} As previously stated, this nozzle is characterized by the fact that in the minor axis plane the hot jet stream is forced by the inner shape of the nozzle towards the conical plug at a steep angle, while in the major axis plane the inner surface of the elliptic nozzle towards the exit has a slope close to zero (see also Figure 1). It was designed with the expectation of producing a jet which would take on a 3:1AR elliptic cross-section downstream of the plug. Figure 7 presents the instantaneous velocity magnitude distributions in the major (x-z) and minor (y-z) axis planes as calculated by LES for the 3:1 AR elliptic plug nozzle (C1@3:1AR). Just downstream of the center-body tip in the major axis plane the jet bifurcates and the two
Figure 5. 2:1 AR elliptic jet case without center-body. Comparisons of normalized time-averaged values of axial velocity component predicted by LES with the experimental data in the major axis plane (a) and minor axis plane (b).

Figure 6. 2:1 AR elliptic jet case without center-body. Comparisons between the LES results and experimental data in terms of jet width at \( W/W_c = 0.5 \) as a function of normalized stream-wise distance.

developed potential regions diverge as the jet spreads.

Figure 8 shows a snapshot of the vortical structures developed in the C10:3:1AR jet, identified by using the \( \lambda_2 \) vortex visualization method. Pairs of vortical rings are visible in the major axis plane just downstream of the nozzle. Similar vortical rings were identified by using LES to occur for 4:1 AR rectangular jets but after an initial axis switching. These studies concerning vortex ring dynamics show that as the bifurcation progresses, global self-deformation also may occur and the higher curvature regions of the vortex at the major axis side move downstream faster leading to a slight inclination of the split rings with respect to the cross-stream plane. As a consequence, the split rings move toward each other and, in turn, the upstream portions of the vortices move away from each other. This repeated dynamics of the split rings will lead eventually to the collision of the vortical rings. Vortex ring bifurcation after the jet initially changed axis was observed also experimentally in elliptic jets with aspect ratios above 3.5. However, in the present study probably due to the plug nozzle design the jet bifurcation occurs just downstream of the nozzle plug without an initial axis switching, the vortical structures having the same orientation as the elliptic nozzle.

The time-averaged velocity magnitude distributions along several cross-planes situated at different sta-
Figure 7. Snapshots of velocity magnitude distributions in the major axis plane (a) and in the minor axis plane (b) as predicted by LES for the 3:1 AR elliptic jet case with center-body (C1@3:1AR).

Figure 8. Instantaneous vortical structures in the flow field as captured by LES for the 3:1 AR elliptic jet with center-body (C1@3:1AR).

tions from the inlet in the flow direction are presented in Figure 9. The presence of the center-body causes splitting of the main jet into two individual jets with their centerlines located in the major axis plane of the elliptical nozzle. Another finding is that both developed potential core regions are not circular in cross-section but tear like shaped.

The normalized mean axial velocity profiles \( \frac{W}{W_o} \) as calculated by LES along major and minor axes at different positions downstream from the tip of the center-body (\( z = 0.48D_{eq}, 1.44D_{eq}, \text{ and } 3.36D_{eq} \)) are compared with PIV data measured for the same jet configuration in Figures 10a and 10b. An overall good agreement is found between the two sets of data. The jet spreading in the major axis plane is obvious as the jet develops downstream of the nozzle. The comparison in terms of normalized axial velocity fluctuations (RMS) is presented in Figures 11a and 11b. In the major axis plane, the four peaks in the RMS values show where the generated shear layers are located and are again an indication of the fact that the primary jet splits into two separated jets. In the minor axis plane the RMS values of the main velocity component show similarities with that of a single stream jet.

The second and the third elliptic plug nozzle geometries (C2@3:1ARnf and C3@2:1ARnf) were designed with the expectation of producing a jet which would take on a 3:1AR and respectively a 2:1AR elliptic cross-section downstream of the center-body and were intended to find how the jet behavior is influenced by not forcing the flow stream towards the conic plug in the minor axis plane. Accordingly, the inner shape of the elliptic nozzle was modified so that in the minor axis plane the exhausting flow stream is not impinging anymore at a steep angle the surface of the conic plug. The time-averaged velocity magnitude fields for both modified configurations are shown in Figure 12. The qualitative comparison with the data obtained for the C1@3:1AR elliptic plug nozzle (see also Figure 9) show important differences in the way in which the jet
spreads when it is not forced to impinge the conical plug in the minor axis. Comparisons between the normalized time-averaged axial velocities as calculated by LES are presented for all three elliptic plug nozzle set-ups (C1@3:1AR, C2@3:1ARnf, and C3@2:1ARnf) in Figure 13, while the corresponded plots of normalized RMS values of axial velocity are depicted in Figure 14. The data were extracted along both major and minor axes at four different positions downstream from the tip of the center-body (0.48\(D_{eq}\), 1.44\(D_{eq}\), 3.36\(D_{eq}\), and 11.0\(D_{eq}\)). It was found that the jet exhausting the elliptic plug nozzle system characterized by forcing in the minor axis the flow stream towards the conic plug (C1@3:1AR) spreads the most in the major axis plane and spreads the least in the minor axis plane. Allowing the flow stream to follow the geometry of the plug in the minor axis plane resulted in increased jet spreading in the minor axis and decreased spreading of the jet in the major axis plane in both C2@3:1ARnf and C3@2:1ARnf cases as compared with C1@3:1AR case.

The cross-sectional development of the jets downstream of the nozzle is emphasized in Figure 15 by plotting the normalized jet width in the major and minor axis planes (\(x/D_{eq}\) and \(y/D_{eq}\)) at \(W/W_c = 0.5\) as a function of normalized stream-wise distance \(z/D_{eq}\). The data show that the axis switching does not occur for any of the configurations analyzed in this study. However, the C3@2:1ARnf nozzle generated a jet that has the largest spreading in the minor axis plane and the lowest in the major axis plane of all considered scenarios.

V. Discussion and Conclusions

The LES in-house solver capabilities in predicting the axis-switching phenomenon for a jet exhausting a 2:1AR elliptic nozzle without conic plug were proved and validations of the results against the experimental data were performed.

The conclusions drawn from the LES calculations of a simple elliptic jet provided support for an investigation into how the introduction of the conic plug affects the elliptic jet development including axis-switching. The study was motivated by the fact that most of the published data on elliptic jets refer to nozzles without conical plug or center-body within their exit planes and important engineering applications like the separate-flow exhaust nozzle systems from jet engines generate jets from nozzles that include a conical plug. Three geometrical configurations of the elliptic plug nozzle were investigated. The first configuration in the study (C1@3:1AR) was designed with the expectation of producing a jet which would take on a 3:1AR
Figure 10. Normalized time-averaged axial velocity plots along major (a) and minor (b) axes at different positions downstream from the tip of the center-body (0.48Deq, 1.44Deq, 3.36Deq) for C1@3:1AR case. LES versus experimental data.

Figure 11. Normalized axial velocity fluctuations plots along major (a) and minor (b) axes at different positions downstream from the tip of the center-body (0.48Deq, 1.44Deq, 3.36Deq) for C1@3:1AR case. LES versus experimental data.
elliptic cross-section downstream of the conic plug. Its geometrical features are that in the major axis plane the inner surface of the elliptic nozzle towards the exit has a slope close to zero, while in the minor axis plane the slope is steeper that it would be for a corresponding round nozzle. This forces the flow stream towards the conic plug in the minor axis plane. The second and the third elliptic plug nozzle geometries (C2@3:1ARnf and C3@2:1ARnf) were intended to find how the jet behavior is influenced by not forcing in the minor axis plane the flow stream towards the conic plug and by changing the aspect ratio of the nozzle.

The presence of the plug causes splitting of the initial elliptic jet into two individual jets with their center-lines located in the major-axis plane of the elliptical nozzle. Previous experimental and numerical studies\textsuperscript{10,19} emphasized the bifurcation phenomenon observed after an initial jet axis-switching for non-circular nozzles with aspect ratios between 3.5 and 12. However, in the present study the bifurcation occurs for all configurations just downstream of the conic plug without axis-switching.

It was found that the spreading of the jet downstream of the elliptic plug nozzle it can be influenced by changing the geometrical characteristics of the elliptic nozzle (i.e. its inner shape or its aspect ratio). In the major axis plane the largest jet spreading was found for the 3:1AR elliptic plug nozzle (C1@3:1AR), while in the minor axis plane the jet exhausting from the 2:1AR elliptic plug nozzle (C3@2:1ARnf) spread the most. The LES results calculated for the 3:1 AR elliptic jet with center-body configuration (C1@3:1AR) were compared with the experimental data obtained for the same set-up using PIV.\textsuperscript{22} A fair agreement was
Figure 13. Normalized time-averaged axial velocity plots along major (a) and minor (b) axes at different positions downstream from the tip of the center-body (0.48D_{eq}, 1.44D_{eq}, 3.36D_{eq}, and 11.0D_{eq}) as calculated by LES. C1@3:1AR versus C2@3:1ARnf versus C3@2:1ARnf.

found between the two sets of data.

References

Figure 14. Normalized axial velocity fluctuations plots along major (a) and minor (b) axes at different positions downstream from the tip of the center-body (0.48\(D_{eq}\), 1.44\(D_{eq}\), 3.36\(D_{eq}\), and 11.0\(D_{eq}\)) as calculated by LES. C1@3:1AR versus C2@3:1ARnf versus C3@2:1ARnf.

Figure 15. LES predictions of jet width as a function of normalized stream-wise distance. C1@3:1AR versus C2@3:1ARnf versus C3@2:1ARnf.

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