1.1 Fluids A Liquids & Gases

Macroscopic perspective:

a) A fluid is a substance that cannot sustain a shear stress, it continuously deforms when subjected to shear.

b) A solid will deform under the application of a shear stress and move to a new equilibrium.
1.1 Fluids A Liquids & Gases

Microscopic perspective:

a) Solids - strong intermolecular attractive forces.

b) Fluids – much weaker intermolecular forces.
   i A liquids occupy a more or less well defined volume,
   ii A gases occupy all the volume attached to them.

Note: Intermolecular attractive forces between liquid molecules is > than between gas molecules.

Incompressible flow A when density ($\rho$) remains constant, regardless of how the flow changes.

Compressible flow A Flows that experience significant density changes, i.e., a gas.
(See the equation of state $P = \rho RT$).

1.2 Review of Fluid Dynamics

6. Basic Concepts and Definitions:

Solids - A substance which has a definite shape regardless of whether small to moderate shear forces are applied to its surface. (Molecules are closely positioned and have large intermolecular forces.)

Fluids - Substances when at rest, cannot sustain a shear force (or tangential force).

• Liquids – A state of matter in which molecules are relatively free to change their position with respect to each other but restricted by intermolecular (cohesive) forces, so as to maintain a relatively fixed volume.

• Gases – A state of matter in which the molecules are practically unrestricted by intermolecular forces (molecules spaced relatively far apart). Hence, a gas has neither a definite shape nor volume.
1.2 Review of Fluid Dynamics

**Body Forces** – those forces which involve action from a distance, and are proportional to either the volume or mass of a body. Examples of body forces are those arising from gravity, magnetic fields, electrodynamics, to name a few.

**Surface Forces** - those forces which are exerted at the control surface by the material outside the control volume on the material inside the control volume. Examples of these surface forces are those arising from:
- normal stresses (or pressure),
- shear stresses (viscous or turbulent),
- surface tension (when interfaces between phases exist).

**Pressure** - force/unit area, exerted perpendicular to a surface.

**Density** ($\rho$) - mass/unit volume; in general $\rho = f(T,P)$

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**Continuum** – When the properties of a fluid are considered to be continuously distributed throughout the region of interest, or when the dimensions of the problem are large w.r.t. the spacing between the molecules.

**Note:**
- i) As the pressure is significantly reduced, the average distance between molecules becomes large compared to the dimensions of the object over which the fluid is flowing. Under these conditions the fluid is now considered a rarified gas & no longer fulfills the continuum assumption.
- ii) There are $2.7 \times 10^{16}$ molecules contained in a cubic millimeter of air at standard conditions (STD).
- iii) To determine if the continuum assumption is valid, compare the characteristic length $\ell$, of the object under study to the molecular mean free path ($\lambda$), which is the average distance a molecule travels before it collides with another molecule.

- If $\ell >> \lambda$, the continuum model is acceptable.
- For air at STD conditions, $\lambda \approx 6 \times 10^{-6}$ cm = 60 nm.
1.2 Review of Fluid Dynamics

• Table A.2 Properties of the U.S. Standard Atmosphere (EE & BG Units)

<table>
<thead>
<tr>
<th>Geometric Altitude (z, ft)</th>
<th>Temp (°R)</th>
<th>Pressure (p, psia)</th>
<th>Density (ρ, lbm/ft³)</th>
<th>Density Acceleration (g, lbf s²/lb)</th>
<th>Viscosity (μ, lbf s/ft²)</th>
<th>Kinematic Viscosity (ν, ft²/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-15000</td>
<td>572.2</td>
<td>24.628</td>
<td>1.162E-1</td>
<td>3.610E-3</td>
<td>32.220</td>
<td>4.031E-7</td>
</tr>
<tr>
<td>-10000</td>
<td>554.3</td>
<td>20.847</td>
<td>1.015E-1</td>
<td>3.155E-3</td>
<td>32.205</td>
<td>3.935E-7</td>
</tr>
<tr>
<td>-5000</td>
<td>536.5</td>
<td>17.554</td>
<td>8.831E-2</td>
<td>2.745E-3</td>
<td>32.189</td>
<td>3.833E-7</td>
</tr>
<tr>
<td>0</td>
<td>518.7</td>
<td>14.696</td>
<td>7.647E-2</td>
<td>2.377E-3</td>
<td>32.174</td>
<td>3.736E-7</td>
</tr>
<tr>
<td>5000</td>
<td>500.8</td>
<td>12.054</td>
<td>6.590E-2</td>
<td>2.048E-3</td>
<td>32.159</td>
<td>3.636E-7</td>
</tr>
<tr>
<td>10000</td>
<td>483.0</td>
<td>10.108</td>
<td>5.648E-2</td>
<td>1.756E-3</td>
<td>32.143</td>
<td>3.534E-7</td>
</tr>
<tr>
<td>20000</td>
<td>474.1</td>
<td>6.759</td>
<td>4.077E-2</td>
<td>1.267E-3</td>
<td>32.112</td>
<td>3.326E-7</td>
</tr>
<tr>
<td>30000</td>
<td>411.8</td>
<td>4.373</td>
<td>2.866E-2</td>
<td>9.097E-4</td>
<td>32.082</td>
<td>3.107E-7</td>
</tr>
<tr>
<td>40000</td>
<td>390.0</td>
<td>2.730</td>
<td>1.890E-2</td>
<td>6.873E-4</td>
<td>32.051</td>
<td>2.969E-7</td>
</tr>
<tr>
<td>50000</td>
<td>390.0</td>
<td>1.692</td>
<td>1.171E-2</td>
<td>3.639E-4</td>
<td>32.020</td>
<td>2.969E-7</td>
</tr>
<tr>
<td>60000</td>
<td>390.0</td>
<td>1.049</td>
<td>7.296E-3</td>
<td>2.566E-4</td>
<td>31.990</td>
<td>2.969E-7</td>
</tr>
<tr>
<td>70000</td>
<td>392.2</td>
<td>0.651</td>
<td>4.479E-3</td>
<td>1.392E-4</td>
<td>31.959</td>
<td>2.969E-7</td>
</tr>
<tr>
<td>80000</td>
<td>397.7</td>
<td>0.406</td>
<td>2.758E-3</td>
<td>8.571E-5</td>
<td>31.929</td>
<td>2.969E-7</td>
</tr>
<tr>
<td>90000</td>
<td>403.1</td>
<td>0.255</td>
<td>1.710E-3</td>
<td>5.315E-5</td>
<td>31.898</td>
<td>2.969E-7</td>
</tr>
<tr>
<td>100000</td>
<td>408.6</td>
<td>0.162</td>
<td>1.068E-3</td>
<td>3.316E-5</td>
<td>31.868</td>
<td>2.969E-7</td>
</tr>
<tr>
<td>150000</td>
<td>479.1</td>
<td>0.020</td>
<td>1.112E-4</td>
<td>3.456E-6</td>
<td>31.716</td>
<td>3.512E-7</td>
</tr>
<tr>
<td>200000</td>
<td>457.0</td>
<td>0.003</td>
<td>1.060E-5</td>
<td>2.570E-7</td>
<td>31.566</td>
<td>3.326E-7</td>
</tr>
<tr>
<td>250000</td>
<td>351.8</td>
<td>0.000</td>
<td>7.034E-6</td>
<td>7.034E-8</td>
<td>31.426</td>
<td>3.272E-7</td>
</tr>
<tr>
<td>300000</td>
<td>332.9</td>
<td>0.000</td>
<td>4.488E-6</td>
<td>4.625E-9</td>
<td>31.271</td>
<td>2.930E-7</td>
</tr>
</tbody>
</table>

Note:

1) \( \rho \) is a function of temperature and pressure.

2) The density of air at STD is \( \rho = 1.23 \text{ kg/m}^3 \) (or 0.00238 slug/ft³) at \( P = 101.33 \text{ kPa} \) (14.696 psia) and \( T = 15^\circ \text{C} \) (59\(^ 5\)F).

Note: Typical satellite in LEO is approx 500 to 500 kilometers.

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1.2 Review of Fluid Dynamics

• Incompressible fluid – a fluid in which the \( \rho \) is assumed constant

Note: Air @ T = constant experiences a 17% density change when P changes from 101kPa to 119kPa.

FIG. 1-A Density of water at 0°C versus pressure. The density is given in terms of the density at 1 atmosphere pressure. (Data from International Critical Tables, McGraw-Hill Book Company, 1959; Courtesy of the National Academy of Sciences, National Research Council, Washington, D.C.)
1.2 Review of Fluid Dynamics

Viscosity — a thermophysical property which represents the resistance to the sliding motion of one fluid layer over another.

- A fluid undergoes a continuous deformation (or strain) when subjected to a shear stress, $\tau$.
- Relating the $\tau$ to the rate of deformation is accomplished using the absolute viscosity, $\mu$, which is a property of the fluid.
- Deformation rate (i.e., strain rate) is the velocity gradient, and in 1D is $dU/dy$.
- Therefore, the 1D shear stress relation.

$$\tau = \mu \frac{du}{dy} \quad (1.4)$$

- Absolute or Dynamic viscosity ($\mu$) $\rightarrow$ lb sec / ft$^2$
- Kinematic viscosity ($\nu$) $\rightarrow$ ft$^2$ / sec or m$^2$/sec $\nu = \frac{\mu}{\rho}$
1.2 Review of Fluid Dynamics

• If the fluid is compressible and if significant changes in volume occur, an additional viscous stress coefficient will be required. This coefficient is called the second or bulk viscosity.

• If \( \dot{\tau} \) is independent of the velocity gradient (i.e., the rate of strain), that is if \( \dot{\tau} \) varies linearly with \( \frac{du}{dy} \) the fluid is called Newtonian.

• Examples of Newtonian fluids are water, air, alcohol, gases and most petroleum products (where \( \mu \) is practically independent of the velocity gradient).

• Note: The absolute viscosity (\( \mu \)) is in general a function of \( P, T \);
  - although changes in \( \mu \) with \( P \) are usually small,
  - changes due to \( T \) may be very large.
  - (see Figures. 3A & 4A and Table 1A.)

---

TABLE 1-A, Displays the effect of pressure on the absolute viscosities of water and a typical lubricating oil (Similar to SAE 30).

The values in the table represent the absolute viscosity (\( \mu \)) at the specified pressure divided by the (\( \mu \)) at 1 atmosphere.

<table>
<thead>
<tr>
<th>Substance</th>
<th>Pressure in Atmospheres</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>100</td>
</tr>
<tr>
<td>Water at 30 °C</td>
<td>1.0</td>
</tr>
<tr>
<td>Water at 10 °C</td>
<td>1.0</td>
</tr>
<tr>
<td>Representative Lubricating Oil (~ SAE 30) at 55 °C</td>
<td>1.45</td>
</tr>
</tbody>
</table>

1.2 Review of Fluid Dynamics

![Image of fluid viscosity graphs]


**FIG. 4-A** Kinematic viscosity of various fluids. $S$ has the same meaning as in Fig. 3-A. Prepared from data in R.L. Daugherty and A. C. Ingersoll, *Fluid Mechanics*, McGraw-Hill Book Company, New York, 1954.

**Note:**
1. Air viscosity increased by 20% as $T$ increased from 18 °C to 100 °C, however the viscosity of H2O decreases by almost a factor of 4 over the same T's.
2. In general gas $\mu$'s increase with $T$ and the viscosity of liquids decrease with increasing $T$.

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1.3 What is Compressibility

- Compressibility is a measure of the relative change in fluid density (or volume) due to a corresponding pressure change.

- Compressibility in gases undergoing low speed motion is not an issue, however as the speed increases so does the effects of compressibility.

- Compressibility in liquids is typically not an issue.

- The thumb rule; $M = 0.3$
1.3 Re-entry Vehicles

**Expansion (Rarefaction) Waves:**
- lowers $p$, $T$ and $P$ of the air
- interacts and weakens bow shock

**Ambient air**

**Boundary Layer:**
- fluid goes to zero at surface
- fluid kinetic energy is converted to thermal energy

**Bow Shock:**
- normal shock standing off leading edge
- conical oblique shock away from leading edge
- acoustic wave in far field

**Detached Shock:**
I- Observer moving w/projectile: shock wave is stationary and flow steady.
II- Observer at rest: shock is moving down and the flow is unsteady.
1.4 Review

SI \text{ A International System} \n\begin{align*}
\text{kilogram A unit of mass} \\
\text{meter A unit of length} \\
\text{second A unit of time} \\
\text{Kelvin A unit of temperature}
\end{align*}

\[ K = ^0C + 273.15 \]

- In this course we will typically express pressure in Pascals \((Pa = 1 \text{ N/m}^2)\) and will only refer to absolute pressures unless otherwise specifically indicated.

- Velocity magnitudes in this system will be labelled as \(u, v, \text{ and } w\) corresponding to the speed components in the \(x, y, \text{ and } z\) directions, respectively.

\[ \dot{V}(x, y, z, t) = u(x, y, z, t)\hat{i} + v(x, y, z, t)\hat{j} + w(x, y, z, t)\hat{k} \]

- In a general sense, flow variables may be functions of the spatial coordinates \(x, y, z, \text{ and time, } t\),

- c.f., the velocity vector or pressure \(P \Rightarrow f(x,y,z,t)\).

\textbf{KINEMATIC PROPERTIES}

- Let \(Q\) represent any property of the fluid \((\rho, T, p, \text{ etc.})\), so the total differential change in \(Q\) is:

\[ dQ = \frac{\partial Q}{\partial x} dx + \frac{\partial Q}{\partial y} dy + \frac{\partial Q}{\partial z} dz + \frac{\partial Q}{\partial t} dt \]

- Spatial increments can be determined by:

\[ dx = u dt \text{ and } dy = v dt \text{ and } dz = w dt \]
1.4 Review

\[ \nabla \times \vec{V} = 0 \]  
If the curl of the velocity field is zero  
\begin{itemize}  
\item Flow is irrotational and  
\item Velocity can also be written as the gradient of a scalar function, \( \phi \)  
\end{itemize}

\[ \vec{V} = \nabla \phi \]

\[ \nabla \cdot \vec{V} = 0 \]  
If the divergence of the velocity field is zero  
\begin{itemize}  
\item Flow is incompressible  
\end{itemize}

\[ \nabla^2 \phi = 0 \]  
If true  
\begin{itemize}  
\item Laplace equation holds  
\end{itemize}

\[ \nabla \times (\nabla q) = 0 \]  
The curl of the gradient of a scalar function is zero

\[ \nabla \cdot (\nabla \vec{A}) = 0 \]  
The divergence of the curl of a vector is zero

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1.4 Review

- **Gradient Theorem**
  - Vector equation involving a scalar function, "q"
  - Limits of integration is set such that surface encloses the volume
  - The unit normal vector, "n" points outward

\[ \iiint \nabla q d\forall = \iint q \hat{n} dS \]

- **Divergence Theorem (Gauss')**
  - The vector quantity is "A"
  - Scalar equation results

\[ \iiint \nabla \cdot \vec{A} d\forall = \iint \vec{A} \cdot \hat{n} dS \]

- **Stokes Theorem**
  - Direction of n is given by right hand rule over the path length "dl"

\[ \iint_{S} (\nabla \times \vec{A}) \cdot \hat{n} dS = \int_{L} \vec{A} \cdot d\vec{L} \]
Material used from:

- Brian J. Cantwell - Stanford University
- Bernard Parent - Pusan National University
- Joseph M. Powers - University of Notre Dame
- Niklas Andersson - Chalmers University of Technology
- Gary Settles - Penn State University
- Others, list will be updated as notes get completed.