Compressible Flow & Thermodynamics

(AEEM 6041)
Fall 2019
Chapter 5:
Applications

5.1 Introduction

• In this chapter engineering applications of compressible flow will be examined.
• In all cases one dimensional flow of an ideal gas, with constant specific heats will be assumed.
• In addition to full Isentropic nozzle flow we will also briefly note the case when the exiting nozzle flow is either:
  i) over-expanded – if the choked flow through a C-D nozzle experiences an $p_e < p_b$
  ii) under-expanded – if the choked flow through a C-D nozzle experiences an $p_e > p_b$
5.1 Introduction

Over-expanded flow: $p_e < p_b$

Under-expanded flow: $p_e > p_b$
5.2 Performance of Converging - Diverging Nozzles

- Gas stored in a reservoir at $p_r$ is allowed to flow (fig 5.1):
  - i) $p_b = p_r$: no flow.
  - ii) As $p_b$ is reduced below $p_r$ subsonic flow is initiated throughout nozzle (curves 1-3).
  - iii) When $p_b$ is reduced to the level represented by curve 4, $M_T = 1$ and the nozzle is said to be choked.
    Further decrease in $p_b$ will not result in any increase in $\dot{m}$.
  - iv) As $p_b$ is further reduced (below curve 4), a normal shock appears in the diverging section of the nozzle, downstream of the throat (curves a & b).
- Only increases in $p_r$ can cause to increase $\dot{m}$.

- If a low enough $p_b$ is set, the shock would position itself @ the nozzle exit plane (curve c).
5.2 Performance of Converging-Diverging Nozzles

- As the static pressure, $p$, decreases in the converging region of the nozzle and $M_T = 1$ (i.e., choked flow), the flow in the diverging region is supersonic with $p$ continuing to drop up to the location of the normal shock (fig 5.2).

- After the shock the flow again becomes subsonic & pressure increases to $p_e$. Since the flow is subsonic at the exit, $p_e = p_b$.

- As $p$ is further lowered to that of curve c a shock positions itself at the nozzle exit ($p_e = p_b$).

- Continuing to lower $p_b$ below curve c, the shock inclined at an angle to the flow appears at the exit and $p_e < p_b$.

- This shock (fig 5.3) is weaker than a normal shock and is referred to as an oblique shock (which results in an non-isentropic deceleration to a lower supersonic speed, curve d).
5.2 Performance of Converging-Diverging Nozzles

- Further reductions in $p_b$ (i.e., $p_{b1} > p_{b2}$) causes the angle between the shock and the flow to decrease, thus decreasing the shock strength until the flow is fully expanded (fig 5.4).

vi) When the flow is fully expanded to $p_b$, the isentropic state is reached (curve 5) and $p_e = p_b$.

vii) When $p_b$ is reduced below that of curve 5 ($p_e > p_b$), a pressure decrease occurs outside the nozzle in the form of expansion waves (curve e and fig 5.5).

Examples of supersonic flow with an arrangement of oblique shock waves beyond the exit plane.
5.2 Performance of Converging - Diverging Nozzles

Note:

1 - Oblique shock waves and expansion waves represent non-one-dimensional flows, and therefore 1D analysis is no longer valid.

2 - For all values of $p_b$ below curve c, the flow adjusts to $p_b$ outside the nozzle, and the flow inside the nozzle remains unchanged, (i.e., velocity and pressure are constant at the nozzle exit plane).

Fig 5.6 presents $p_e$ as $f(p_b)$:

State I - $M_e < 1$, $p_e = p_b$ & points 1 to 4 & a to c.
State II - $M_e > 1$, $p_e = p_b$ & point 5 (perfect isentropic expansion).

State III - $M_e > 1$, two additional conditions can exist, pts d & e:

a) Between curves c, d, & 5: $p_e < p_b$, the nozzle is over-expanded.

b) Below curve 5 & $p_e > p_b$, the nozzle is under-expanded.
Example 5.1:
A converging-diverging nozzle is designed to operate with an $M_E = 1.75$. The nozzle is supplied by an air reservoir at 5 MPa. Assuming 1D steady flow calculate:

a) Maximum $p_B$ to choke the nozzle.

b) Range of $p_B$ over which a normal shock will appear in the nozzle.

c) Value of $p_B$ necessary for the flow to be perfectly expanded to the design $M$ #.

d) Range of $p_B$ for supersonic flow @ the nozzle exit plane.

Solution:
Example 5.2:

A rocket nozzle is designed to operate supersonically with a combustion chamber pressure of 3 MPa and an ambient pressure of 101 kPa. Find the ratio between the thrust at sea level to the thrust in space (0 kPa). Assume a constant chamber pressure with the chamber temperature of 1600 K. Also assume the rocket exhaust gases behave as an ideal gas with $\gamma = 1.3$ and $R = 0.40 \text{ kJ/kg K}$.

Solution:
5.3 Supersonic Wind Tunnels

- In order to provide isentropic deceleration and complete pressure recovery after the test section of a supersonic wind tunnel, a second throat after the test section is required (fig 5.12).

![Diagram 5.12](image)

> $M = 1$
> $p_i$
> Nozzle
> Test section
> $M = 1$
> $p_e = p_B$
> Supersonic diffuser

- To initiate flow, a pressure difference must be maintained across the entire system. Mass begins to flow as $p_B/p_i$ is lowered from 1.0. Initially the flow is subsonic in the test section (fig 5.13).

![Diagram 5.13](image)

- As $p_B/p_i$ is further decreased a shock forms at the throat of the supersonic nozzle and moves to the nozzle exit.

- Once the shock is at the test section entrance any further decrease in $p_B$ will cause the shock to jump through the test section into the supersonic (SS) diffuser.

![Diagram 5.14b](image)
5.3 Supersonic Wind Tunnels

- To obtain supersonic flow in the test section the shock must also pass through the 2nd throat. For this to occur the cross sectional area of the diffuser must be at least $A_i^*$ (or the flow will choke in the supersonic diffuser and not allow the full mass flow to pass).

- The worst case occurs when the loss in stagnation pressure is maximum and a normal shock appears in the test section.

- An examination of the normal shock tables indicates at $M=1$,
  \[
  \frac{p_{t_2}}{p_{t_1}} = 1.0
  \]
  so no loss in stagnation pressure.

- However this stagnation pressure loss increases as $M#$ increases.

- Therefore it is more efficient if the shock wave moves out of supersonic test section, into a lower $M#$ region and positions itself at the 2nd throat of the SS diffuser.

- If $A_{i_2} < A_2^*$ the flow will be restricted and the shock will not be able to reach the test section, under these conditions supersonic flow cannot be achieved in the test section.

- For example, if the test section $M#$ = 2.0, from the normal shock tables the ratio is.
  \[
  \frac{p_{t_2}}{p_{t_1}} = 0.7209
  \]

- Recalling Eq 4.15:
  
  rearrange,
  \[
  \frac{p_{t_1}}{p_{t_2}} A_{i_1}^* = \frac{A_{i_2}^*}{A_2^*}
  \]
  \[
  \frac{p_{t_1}}{p_{t_2}} = \frac{A_{i_2}^*}{A_2^*} = 0.7209
  \]

- So from continuity the throat area of the supersonic diffuser required to pass the shock is
  \[
  A_{i_2}^* = \frac{A_{i_1}^*}{0.7209} = 1.39 A_{i_1}^*
  \]
5.3 Supersonic Wind Tunnels

Hence, during start-up the area of the 2nd throat must be at least 39% larger than the 1st throat, for $M = 2.0$.

If $p_B/p_i$ is further lowered, the shock wave now jumps into the diverging portion of the supersonic diffuser, and the shock is said to be "swallowed" by the diffuser.

After start up and "swallowing" the shock, the pressure recovery that occurs in the SS diffuser can be obtained.

This is represented by the pressure ratio $p_B/p_i$, which can be increased by moving the shock back upstream to the SS diffuser throat, the position at which the shock strength is minimum (Fig 5.17).

Fig 5.17 represents the optimum operating condition for a fixed geometry supersonic diffuser.
5.3 Supersonic Wind Tunnels

- In reality, there is still a loss in stagnation pressure across the shock at the diffuser throat. This loss must be made up by a compressor (this loss is not accounted for in the normal shock tables).

- A variable area diffuser throat can also be employed. After the initial shock has been swallowed the diffuser throat area can be reduced and therefore again raise the overall pressure ratio $\frac{p_B}{p_1}$.

- Ideally, this means moving the shock to the diffuser throat, where $M = 1$ and the shock is of vanishing strength.

- In addition to a compressor, supersonic nozzle and supersonic diffuser, secondary equipment such as air filters and dryers are also required to operate a supersonic facility.

Note:

a - Filters remove lubrication oils and dirt from the compressed air.

b - An air dryer removes water vapor from the air, and this minimizes the chance of condensation, which can occur at the low static temperatures encountered in a supersonic test section.

Condensation of water can lead to shock waves, and possible damage to measurement transducers and/or the test model.

c - At higher $M$'s a heater may also be required to raise the $T_r$ of the inlet air, thereby preventing condensation & liquefaction of gases in the test section (oxygen & nitrogen).

If a heater is impractical, a gas with a low boiling point (e.g., helium) may be used.
5.3 Supersonic Wind Tunnels

Example 5.3:
A continuous supersonic wind tunnel is designed to operate a 25cm diameter test section at $M = 2$, with static conditions duplicating those of air @ 20 km: $p = 5.53$ kPa & $T = 216.7$ K

![Diagram](Image)

**Figure 5.19a**

A fixed geometry supersonic diffuser is located downstream of the test section. Assume an isentropic compressor with a cooler located between the compressor and nozzle, so the compressor inlet temperature is set to be the test section $T_t$.

Neglecting friction and boundary layer effects, determine the compressor power needed for startup & steady state operation.

Solution:
- During
5.3 Supersonic Wind Tunnels
5.4 Converging-Diverging Supersonic Diffusers

- The forward thrust of an air-breathing jet engine is provided by the acceleration of the exhaust gases.
- In turbojet & ramjet engines, air is compressed & energy is added by the combustion process, and finally the hot combustion gases are ejected at high velocity from the exhaust nozzle.
- In flight the high velocity air must be decelerated prior to being ingested into the compressor or combustion zone.

- The static pressure rise achieved during this deceleration is very important to engine operation. The pressure recovery of the entering air determines to a large extent the magnitude of the nozzle exhaust velocity.
- The maximum pressure that can be achieved in a diffuser is the isentropic stagnation pressure. It is highly desirable to operate under shock-free isentropic flow.
- If a diffuser is operated @ the design $M_D$ and friction is neglected there is ideally no loss in $p_v$, which is a very desirable condition. But off - design performance can be a problem.
- Lets assume a supersonic diffuser is designed for use at $M_D = 2$, then from the isentropic tables, $\frac{A_{Inlet}}{A_{Throat}} = 1.688$.
5.4 Converging-Diverging Supersonic Diffusers

- If however the $M < M_D$, as noted in the isentropic tables, which indicates that the throat area is not large enough to handle the designed flow.

\[ \frac{A_f}{A_t} < 1.688 \]

- Under these conditions some of the flow must be bypassed around the diffuser. A normal shock positions itself in front of the diffuser, with subsonic flow after the shock.

- Able to sense the presence of the inlet, an appropriate amount of flow "spills over" or bypasses, the inlet.

As the flight $M \#$ increases the normal shock moves toward the inlet lip.

During start-up (even @ $M_D$) with a normal shock located in front of the diffuser, some of the flow must still be bypassed, since $A_f < A_t$, the diffuser is still not able to handle the entire subsonic flow after the shock (fig b).
5.4 Converging-Diverging Supersonic Diffusers

As $M$ is increased above $M_D$, the shock moves to inlet lip (fig c).

- A further increase in $M_D$ causes the shock to reach a new equilibrium position in the diverging portion of the diffuser; in other words the shock is “swallowed” (fig d).

- Once the shock has been swallowed (fig d), a decrease in the Mach number causes the shock to move back toward the throat (fig e), where it reaches an equilibrium position for $M = M_D$.

- At this position the shock is of vanishing strength ($M = 1$). So ideally no loss in stagnation pressure occurs at the design condition.
5.4 Converging-Diverging Supersonic Diffusers

- It is desirable to operate with the shock slightly past the throat, because operation at the design condition is unstable.
- A slight drop in Mach number results in the shock moving back out in front of the inlet. This condition would require an over speed to once again swallow the shock.

\[
\frac{A_1^*}{A_2^*} = 0.7209
\]

A second method for swallowing the shock uses a variable throat area. With the shock in front of the diffuser, the throat area would be increased, allowing more flow to pass through the inlet and bring the shock closer to the inlet lip.

To swallow the shock the throat area would have to be even larger, than that required to accept the flow with a shock positioned at the inlet lip at \( M_D \).

That is slightly larger than \( A_1^* \) with a normal shock at \( M_D \). As shown before if \( M_D = 2.0 \) then;

\[
\frac{A_1^*}{A_2^*} = 0.7209
\]

or \( A_2^* \) must be 39% > \( A_1^* \) to swallow the shock.

Then, once the shock is swallowed the area can be reduced to reach the design point.
Example 5.4:
A supersonic converging - diverging diffuser is designed to operate at $M = 1.7$.
What should the inlet M # to be accelerate the flow in order to swallow the shock during start-up.

Solution:

--- The End ---
5.2 Performance of Converging - Diverging Nozzles

- Subsonic flow at nozzle exit
- Supersonic flow at nozzle exit

No shock
Shock in nozzle

Sonic flow at throat