6.1 Introduction

- In previous chapters, a compression shock normal to the flow was studied. However, there are many physical situations where a compressive shock wave is formed, inclined to the flow. Such a wave is called an oblique shock.
6.1 Introduction

Major Differences between Normal and Oblique Shock Waves

<table>
<thead>
<tr>
<th>Shock Characteristics</th>
<th>Normal Shock</th>
<th>Oblique Shock</th>
</tr>
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<tbody>
<tr>
<td>Occurs in Flows</td>
<td>( M_1 &gt; 1 )</td>
<td>( M_1 &gt; 1 )</td>
</tr>
<tr>
<td>Shock Orientation to Incident Flow</td>
<td>Perpendicular</td>
<td>Inclined to flow</td>
</tr>
<tr>
<td>Flow Deflection angle</td>
<td>No change in flow direction</td>
<td>Flow changes in orientation</td>
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<tr>
<td>Flow Downstream of Shock</td>
<td>Subsonic ( (M_2 &lt; 1) )</td>
<td>Typically ( M_1 &gt; 1 ), but can be ( M_2 &lt; 1 )</td>
</tr>
<tr>
<td>Shock Strength</td>
<td>Large</td>
<td>Low</td>
</tr>
</tbody>
</table>

**Note:** Flow through Shock Waves

1. Entropy increases (Non-isentropic process)
2. Stagnation temperature constant

- Oblique shock waves also represent a sudden, almost discontinuous change in fluid properties, with the shock process itself being adiabatic.
- The flow must undergo a compression.
- Oblique shock wave strength can be consider either strong or weak.
6.1 Introduction

• A simple case is that of a supersonic flow about a 2-D wedge with its axis aligned parallel to the flow direction.

• For small wedge angles, the flow adjusts by means of an oblique shock wave, which is attached to the apex of the wedge.

• After the shock, the flow is uniform and parallel to the wedge surface (fig 6.3).

• The entire flow having been turned through the wedge half-angle $\delta$.

6.2 Equations of Motion of an Straight Oblique Shock Wave

- A simple case is that of a supersonic flow about a 2-D wedge with its axis aligned parallel to the flow direction.

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- The entire flow having been turned through the wedge half-angle $\delta$. 
6.2 Equations of Motion of an Straight Oblique Shock Wave

Selecting the following control volume and write the equations of motion for a uniform supersonic flow over a fixed wedge.

Using the control surface indicated in the above figure, the continuity eq reduces to,

\[ \int_{s} \rho(\vec{V} \cdot d\vec{A}) = 0 \]  \hspace{1cm} (1.9)

and becomes

\[ \rho_1 V_{1n} A_1 = \rho_2 V_{2n} A_2 \]

Since the shock is very thin, \( A_1 = A_2 \)

So,

\[ \rho_1 V_{1n} = \rho_2 V_{2n} \]  \hspace{1cm} (6.1)

The steady flow Momentum Eq is

\[ \sum \vec{F} = \int_{s} \vec{V}(\rho \vec{V} \cdot d\vec{A}) \]  \hspace{1cm} (1.14)
6.2 Equations of Motion of an Straight Oblique Shock Wave

But flow is 2-D, so only two momentum component eqs are required;

The normal component is

\[
-\left[\left(-p_1 A_1\right) + p_2 A_2\right] = V_{1n} \left(-\rho_1 V_{1n} A_1\right) + V_{2n} \left(\rho_2 V_{2n} A_2\right)
\]

\[
p_1 A_1 - p_2 A_2 = \rho_2 A_2 V_{2n}^2 - \rho_1 A_1 V_{1n}^2
\]

Again using the thin shock assumption, \(A_1 = A_2\)

\[
p_1 - p_2 = \rho_2 V_{2n}^2 - \rho_1 V_{1n}^2
\]

(6.2)

Since there is no change in pressure in the tangential direction,

\[
0 = \int_s V_t \left(\rho \vec{V} \cdot d\vec{A}\right)
\]

6.2 Equations of Motion of an Straight Oblique Shock Wave

Recall:

\[
\vec{V} = V_n \hat{i} + V_t \hat{j} + 0 \hat{k}
\]

\[
\vec{A} = A_t \hat{i} + 0 \hat{j} + 0 \hat{k}
\]

\[
V_{1t} \left[\left(-\rho_1 A_1 V_{1n}\right)\right] + \left[\rho_2 A_2 V_{2n}^2\right] V_{2t} = 0
\]

\[
V_{1t} \left(\rho_1 A_1 V_{1n}\right) = V_{2t} \left(\rho_2 A_2 V_{2n}\right)
\]

Recall for \(A_1 = A_2\) the continuity eq (6.1.) becomes,

\[
\rho_1 V_{1n} = \rho_2 V_{2n}
\]

Therefore,

\[
V_{1t} = V_{2t}
\]

(6.3)

Similarly, the energy equation for an adiabatic steady flow simplifies to,

\[
h_1 + \frac{V_{1t}^2}{2} = h_2 + \frac{V_{2t}^2}{2}
\]
6.2 Equations of Motion of an Straight Oblique Shock Wave

or

\[
\begin{align*}
\frac{h_1}{2} + \frac{V_1^2}{2} + V_1V_{nt} &= \frac{h_2}{2} + \frac{V_2^2}{2} + V_2V_{nt}, \\
\text{since} \quad V_{1t} &= V_{2t}, \\
\frac{h_1}{2} + \frac{V_1^2}{2} &= \frac{h_2}{2} + \frac{V_2^2}{2}.
\end{align*}
\]  

Note:

a) Eqs 6.1, 6.2, & 6.4 contain normal velocity components only and are the same as eqs 4.1, 4.2, & 4.3 for a normal shock wave.

b) The pressure ratio, temperature ratio, etc., across an oblique shock wave can be determined by first calculating \( M_1 \) normal to wave and then use the normal shock relations.

---

### Example 6.1:

A uniform supersonic airflow traveling at Mach 2 passes over a wedge (Fig 6.4). An oblique shock, making an angle of 40 degrees with the flow direction, is attached to the wedge under these flow conditions.

The static pressure and temperature in the flow are, 20 kPa and -10°C, respectively.

Determine the static pressure and temperature behind the wave, the \( M_\# \) of the flow passing over the wedge, and the wedge half - angle.

\[ M_1 = 2.0 \]
6.2 Equations of Motion of an Straight Oblique Shock Wave

- In most cases the wave angle is not known, but rather $M_1$ and $\delta$ appear as the independent variables.

- Therefore

  \[ M_{1n} = M_1 \sin \theta \]  
  \[ M_{1t} = M_1 \cos \theta \]  
  \[ M_{2n} = M_2 \sin(\theta - \delta) \]  
  \[ M_{2t} = M_2 \cos(\theta - \delta) \]

- Let's plot, for example $\theta$ vs. $\delta$ for a given value of $M_1$.

- From Eqs 6.5 & 6.6, $M_{1n}$ and $M_{1t}$ can be found for given values of $M_1$ and $\theta$.

- Then for a given $M_{1n}$ and going to the normal shock tables, $M_{2n}$ can be determined.
6.2 Equations of Motion of an Straight Oblique Shock Wave

- Since \( V_{1t} = V_{2t} \) and \( V = M\sqrt{\gamma RT} \), then \( \frac{T_2}{T_1} = \frac{M_{2t}}{M_{1t}} = \sqrt{\frac{T_2}{T_1}} \).
- \( \frac{T_2}{T_1} \) can be found from the normal shock tables for a given \( M_{1n} \).
- With \( M_{2n} \) and \( M_{2t} \) computed, the two unknowns \( M_2 \) and \( \delta \) are calculable from Eqs. 6.7 & 8.

- \( \delta \) vs. \( \theta \) is plotted for a given \( M_1 \).
- \( M_2 \) is plotted vs. \( \delta \) for a given value of \( M_1 \).

Example: Results for \( M_1 = 2.0 \) are shown in Figs 6.5 & 6.

Detailed oblique shock charts are also available.
6.2 Equations of Motion of an Straight Oblique Shock Wave

**Weak Shocks:**
- Smaller $\theta$
- $\theta_{\text{Min}} = \mu$
- $M_2 > 1$ Oblique Shock Wave

**Strong Shocks:**
- $\theta_{\text{Max}} = 90^\circ$
- $M_2 < 1$ always
- Normal Shock

**Mach Waves:**
- $\delta = 0E(\text{deflection})$
- $M_2 = 1$; move at the speed of sound in the medium.

For a given $M_1$ and $\delta$ two solutions are possible. If the solution exists, there may be a weak oblique shock solution with $M_2$ either supersonic or slightly less than 1, or a strong shock solution with $M_2$ subsonic.

For the strong shock solution, the wave makes a large angle with the oncoming flow ($\theta$ close to 90 degrees); for the weak shock, the wave angle is much less. (Fig 6.7).
6.2 Equations of Motion of an Straight Oblique Shock Wave

- In both cases the supersonic flow is turned through the same angle, yet the characteristics of the oblique shocks are quite different.
- A weak shock is accompanied by a relatively small pressure ratio, while the strong shock case results in a large pressure ratio.
- The normal shock represents the limiting case of a strong oblique shock, whereas the limiting case of a weak oblique shock, $\delta = 0$ degrees is isentropic flow.
- Another characteristic of the oblique shock eqns is that for a large enough turning angle, no solution is possible.

In this case the shock stands in front of and detached from the body(Fig 6.8).

6.2 Equations of Motion of an Straight Oblique Shock Wave

- The detached shock is curved (Fig 6.8) with the shock strength diminishing progressively from that of a normal shock at the apex of the wedge to a Mach wave far from the body.
- Shape of the wave, and the shock-detachment distance are dependent on the M # and the body shape.
- Flow over the body is subsonic in the vicinity of the wedge apex, where strong oblique shocks occur, and M # > 1 farther back along the wedge, where the weak oblique shocks are present.
6.2 Equations of Motion of an Straight Oblique Shock Wave

- A detached oblique shock can also occur when supersonic flow approaches a concave corner (Fig 6.9).
- If the turning angle is too great, a solution from the oblique shock charts cannot be found.
- The characteristics of this shock are exactly the same as those of the upper half of the detached shock (Fig 6.8).
- What’s observed is that near the wall the flow after the shock is subsonic, but supersonic out in the flow.

Example 6.2: A uniform flow at $M_1 = 2.0$ passes over a wedge of 10 degree half-angle. Using the oblique shock charts, find $M_2$, $\frac{p_2}{p_1}$, $\frac{T_2}{T_1}$, $\frac{p_{2t}}{p_{1t}}$ and the half-angle above which the shock will become detached.

Let $\gamma = 1.4$
6.2 Equations of Motion of an Straight Oblique Shock Wave

--- The End ---

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Example 6.3: A 2-D supersonic inlet is designed to operate at $M = 3.0$. Two possibilities will be considered; case a) the compression and slowing down of the flow takes place through a single normal shock, and case b) a wedge-shaped diffuser, the deceleration occurs through two weak oblique shocks, followed by a normal shock. The wedge turning angles are each 8 deg. Compare the loss in stagnation pressure for the 2 cases.
6.2 Equations of Motion of an Straight Oblique Shock Wave
6.2 Equations of Motion of an Straight Oblique Shock Wave

6.3 Oblique Shock Reflections

- When a weak two-dimensional oblique shock impinges on a plane wall, a reflected wave is required to straighten the flow, since the flow can not penetrate the wall (Fig 6.11).

- That is if an incident shock wave is deflected toward a wall, a reflected oblique shock wave must be created to deflect the flow back through the same angle and restore the flow direction parallel to the wall.

- The reflected shock is weaker than the incident shock, $M_2 < M_1$. 

![Figure 6.11](image-url)
6.3 Oblique Shock Reflections

Example 6.4:
For a flow at $M_1=2.0$ and an resulting incident oblique shock wave, $\theta = 40$ deg.
Determine the angle of the reflected wave, $\theta_r$, $M_2$, and $M_3$.

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The End

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6.3 Oblique Shock Reflections

- If $M_2$ is low enough, a simple shock reflection maybe not possible for a given $M_2$. That is the required turning angle maybe so great that no solution exists from Figs C.1 & C.2.

- In this case, a Mach reflection occurs (Fig 6.12) which is a strong curved oblique shock forms in the stream, and extends from “O” to the wall at “W”.

- The shock must be normal to wall to prevent the possibility of any flow deflection (satisfying the wall boundary condition at this point).

- Flow after the curved shock “OW” is subsonic and adjusts smoothly to the presence of the wall.

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6.3 Oblique Shock Reflections

- Flow after the curved shock “OW” is subsonic.

- A weak oblique shock “OR” also appears with supersonic flow after the shock.

- The combination of $M > 1$ and $M < 1$ flow after the waves makes an analysis of the Mach reflection extremely difficult and beyond the scope of this course.
6.4 Conical Shock Waves

- Supersonic flow about a right circular cone, while more complex than that about a wedge, bears many similarities to wedge flow.

- For a cone at zero angle of attack with the freestream, a conical shock is attached to the apex of the cone for small cone angles.

- Comparison of wedge & cone flows (Fig 6.13).

**Figure 6.13**

- Wedge, straight parallel flow exists before & after the oblique shock.
- 3-D semi-infinite cone, streamlines after conical shock are curved.
- For an axisymmetric flow about a semi-infinite cone, with no characteristics length along the cone surface, conditions after the shock are dependent only on the conical coordinate $T$.
- Along each line of constant $T$, the flow pressure, velocity, etc. are constant. This also indicates the pressure on the surface of the cone after the shock is constant, independent of distance from the cone apex.
6.4 Conical Shock Waves

- At each point on the conical wave, the oblique shock eqs that were already presented are valid.
- The conical flow behind the wave is isentropic, with the static pressure increasing to the cone surface pressure.
- A solution for the conical shock thus requires fitting the isentropic compression behind the shock to the shock eqs already derived.
- Results shown in Figs C.3, 4, and 5 depict the variation of the shock wave surface pressure coefficient and surface M# with cone semivertex & M.

6.4 Conical Shock Waves

- Only the weak shock solution on a conical body is observed.
- For wedge flows at a large enough - cone angle, there is no solution; the shock stands detached from the cone.
- If we compare wedge & cone solutions it can be seen from the conical shock charts, C.3 4, & 5, that for a given body half-angle and M₁, the shock on the wedge is inclined at a greater angle to the flow direction than the shock on a cone.
- This indicates that a stronger compression takes place across the wedge oblique shock. In other words, the wedge presents a greater flow disturbance then a cone.
- From a physical standpoint, the flow is unable to pass around the side of 2-D wedge since it extends to infinity in the 3rd direction.
6.4 Conical Shock Waves

Example 6.5: Uniform supersonic flow at $M = 2.0$ and a $p=20$ kPa passes over a cone of semi vertex angle, $\theta = 10^\circ$ aligned parallel to flow direction. Determine the shock wave angle, $M^\#$ of the flow along the cone surface, and surface pressure.
6.4 Conical Shock Waves