Scales of Turbulent Motions

• A turbulent flow field is composed of vortical motions (structures or eddies) of different sizes or scales.

• This wide range of length scales is bounded from above by the dimensions of the flow field and from below by the diffusive action of molecular viscosity.

• In an Eulerian system (fixed control volume) the passage of vortices can be seen as a series of waves.

• As such, frequency or wave number is often used to describe the physics of turbulent flows.

• Turbulent Kinetic Energy (TKE) $m^2/s^2$ represents the turbulent energy in the flow.

• Therefore a convenient approach when studying turbulence is to utilize spectral techniques, such as the energy spectrum, $E(k)$.
Scales of Turbulent Motions

- The following figure displays the kinetic energy of the turbulence, \( E(k) \) for the different scales of motion (or wavelengths).

![Image of kinetic energy figure]

\[
TKE = \int_0^\infty E(k) \frac{dk}{[1/m]} = \frac{1}{2} \left( u'^2 + v'^2 + w'^2 \right) \approx \frac{3}{2} u'^2
\]

Note:

a) Where \( u' \) represents the velocity of the large scale turbulent motions (i.e., integral scales) which are \( 0( \ell ) \).

These eddies are directly generated by shear of the mean flow and are referred to as the energy containing eddies.

b) The wave number \( (k) \) represents the eddy size,

\[
k = \frac{2\pi}{\text{wavelength}} \approx \frac{\mathcal{O}}{U} \approx \frac{1}{\text{eddy size}}
\]

where \( \mathcal{O} \) represents the vorticity of a particular scale or motion.

c) Turbulent eddies (i.e., scales) are grouped as follows:
Integral Length Scale

- An estimate of a length scale of these larger eddies is related to the following:
  - Eddy size is $O(\ell)$ and it has an associated characteristic velocity, $u$ and a time scale,
  - Their characteristic velocity ($u$) is of the order of the rms turbulent velocity fluctuation.
  - The energy of an eddy is dissipated in one time scale $t_t$.
  - The Reynolds number associated with the large eddies is referred to as the turbulence Reynolds number,

$$Re_\ell = \frac{u\ell}{v}$$

Taylor Microscale

- The Taylor microscale resides in a wavelength range between the large eddies, and the small eddies.
- This range is referred to as the ____________ and if the $Re_\ell$ is large, these turbulent scales are independent of both the large and small scale motions.
- This region is characterized by the amount of energy that is transported through the energy spectrum/unit time and size of eddy.
- A common Reynolds # used to characterize the turbulence within this range is

$$Re_\lambda = \frac{u\lambda}{v}$$
Kolomogorov Microscale

- This is the smallest turbulent motion (or scale) present within a flow and it’s the scale at which energy is dissipated by molecular viscosity.

- The Reynolds # used to characterize this scale is:

\[ \text{Re}_\eta = \frac{v\eta}{\nu} \]

where \( \eta \) is the length scale and \( \nu \) the velocity scale;

- The Kolomogorov length (\( \eta \)), velocity (\( \nu \)), and time scales (\( t \)) can be written in terms of the rate of energy dissipation (\( \varepsilon \));

\[ \eta = \left( \frac{\nu^3}{\varepsilon} \right)^{1/4} \]

- When \( \text{Re}_\eta = 1 \), energy dissipation due to viscous processes is on the same order as those due to inertia.

Energy Transfer

- Through inertial interactions the large scale motions become unstable and breakup, transferring their energy to smaller and smaller scales of turbulent motions.

- This process is represented by the nonlinear terms in the eqns of motion.
Energy Transfer

- Inertial interactions continue until the length scales become smaller and smaller (i.e. larger and larger velocity gradients) and are on the order of the viscous length scales.
- This is where small scale motions are stable and molecular viscosity becomes effective at dissipating the kinetic energy.

![Energy Transfer Diagram]

- At these small scales the kinetic energy of the turbulence is converted into internal energy, i.e. heat. This process of results in a local increase in temperature.

Energy Cascade

- This process is analogous to a cascade of waterfalls, where each fall breaks down into smaller and smaller falls as the process continues.

![Energy Cascade Diagram]
Energy Cascade

• L.F. Richardson ("Weather Prediction by Numerical Process." Cambridge University Press, 1922) summarized this in the following often cited verse:

  Big whirls have little whirls
  Which feed on their velocity;
  Little whirls have lesser whirls,
  And so on to viscosity

Drawing by Leonardo da Vinci (1452-1519), who recognized that turbulence is made up of eddies of various scales.

Energy Cascade

• As large eddy breakdown occurs due to inertial interactions, the decay of the small scale motions, which is due to viscous effects, become more and more important as the turbulent scales approach zero.

• In the fully developed state it is not the largest eddies that will have the maximum kinetic energy but the eddies in the higher frequency or wavenumber range,

  Small scale \Rightarrow \text{ small wavelength}\n  \text{small wavelength} \Rightarrow \text{large wavenumber}\n  \text{large wavenumber} \Rightarrow \text{large frequency}
Energy Containing Range

- Within this range are the large eddies, intense turbulent motions that were directly generated by the shear in the mean flow.

- Approximately 80% of the turbulent kinetic energy (TKE) is contained in this low frequency range, where the eddies, designated as the energy containing eddies make the main contribution to the total TKE of the turbulence. (i.e., a maximum in the energy spectrum is observed).

- These eddies reside in a size range from $6t > l > l/6$ which is $> t_{EI}$ and is called the energy-containing range.

- Although the more permanent, largest eddies (very low frequency, size ~ L) contain much less energy than the energy containing eddies, their energy is by no means negligible. They may contain as much as 20% of the TKE.
Energy Containing Range

- Analysis of the spectra can further show that these eddies spend approximately 90% of their total lifetime, $t_e$ in the production range. So that once eddies enter the inertial subrange only 10% of $t_e$ remains before its energy is dissipated.

- The lifetime for a large scale eddy, from birth through break up to dissipation or death can be written in terms of it’s lifetime,

- These eddies are highly anisotropic, depend on the conditions of formation, however molecular viscosity has little effect.

- Isotropic - no preference to direction.

Note: Turbulent kinetic energy (TKE) is also written as $K$

Universal Equilibrium Range

- The size of the turbulent motions in this range are $< \ell_{EI}$ (i.e., size of larger eddies).

- Since the time scale of these eddies is small compared to $t_e = \ell_e/u$ the eddies in this range can quickly adapt to changes within the flow and therefore, maintain dynamic equilibrium with the energy transfer by the large scale motions.

- Kolmogorov hypothesis of local isotropy states,

"at sufficiently high Reynolds number, the small scale turbulent motions are statistically isotropic."

Note: $\ell$ is the integral scale of the turbulence, $\ell_e$ is the Kolmogorov scale.
Universal Equilibrium Range

- Kolmogorov hypothesis of local isotropy states, "at sufficiently high Reynolds number, the small scale turbulent motions are statistically isotropic."

- Although the large scales may be anisotropic, there is local isotropy of the small scales. Therefore the small scale motions are independent of the mean flow and the boundary conditions.

- In homogeneous turbulence, the energy is the same everywhere in the flow.

- In isotropic turbulence eddies have the same behavior in all directions:

- This range is subdivided into:
  - Inertial subrange (Taylor Microscale)
  - Dissipation range (Kolmogorov Microscale)

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Universal Equilibrium Range

- The following is meant to demonstrate the instantaneous energy cascade that occurs within a turbulent boundary layer.

A state of universal equilibrium is reached when the rate of energy received from larger eddies is nearly equal to the rate of energy the smallest eddies dissipate into heat.
Inertial Subrange

- Turbulent motions within the inertial subrange are represented by the Taylor microscale ($\lambda$), and reside between $\ell_{EI} > \lambda > \ell_{DI}$.

- Although the size of the turbulent motions in this intermediate range are $< \ell_{EI}$, they still have a sufficiently large Re and as such, are not significantly affected by viscosity.

For eddies in the inertial subrange the rate of energy dissipation ($\varepsilon$) can also be determined by: $\varepsilon$

Kolmogorov also postulated,

"in every turbulent flow at sufficiently high Reynolds number, the statistics of the motions of scale $\lambda$, in the range $\ell >> \lambda >> \eta$ have a universal form that is uniquely determined by dissipation ($\varepsilon$) independent of viscosity ($\eta$)."
Dissipation Range

- Turbulent motions within the dissipation range are represented by the Kolmogorov microscale ($\eta$) which are eddies $< \lambda = \ell_{DI}$.
- One rule of thumb uses the end of this range as: $\ell_{DI} \approx 60\eta$
- Approximately 90% of the energy dissipation occurs in a range of eddy length scales from: $\eta^1$ (i.e., larger than $\eta$)

- Since most of the dissipation occurs at scales that are $> \eta$, the Kolmogorov scale should be interpreted as a measure of the smallest eddy that is present in a turbulent flow at high Re.

- Small scale motions within this range only depend on the rate at which energy is supplied by the large scale motions and the rate at which energy is dissipated ($\varepsilon$: $m^2/s^3$) by molecular viscosity.
Summarizing Length Scales

- Integral length scale:
  (large scale eddies) $\sim \left( \frac{k^{3/2}}{\varepsilon} \right)$

- Taylor Microscale:
  (length scale as turbulence moves to the isotropic state) $\sim (15\nu^{1/2}/\varepsilon)^{1/2}$

- Kolmogorov Microscale:
  (length scale of the smallest eddy) $\sim \left( \frac{\nu}{\varepsilon} \right)^{1/4}$

Using Dimensional analysis Kolmogorov (1941) showed that the energy in the inertial subrange follows;
Energy Spectrum: An Alternate Form

- Experimentally measured 1D spectra (1 velocity component, indicated by "1" and "11" subscripts). The number after each reference denotes the measured $Re_U$, see Pope, pg 235).

- Accurate determination of the energy spectrum requires the simultaneous measurements of all velocity components at multiple points, which is usually not possible.

- However, it is common to record a single velocity component time series at a single spatial location.

- Signal is then converted to the spatial domain using $x = Ut$ (commonly referred to as Taylor’s frozen turbulence hypothesis). It is only valid for $u'/U << 1$, which is not always the case.