Chapter 8  Momentum Transfer in an External Turbulent Boundary Layer

8.1  Transition from a Laminar to Turbulent Boundary Layer
An external laminar boundary layer or laminar pipe flow is found to be unstable in the presence of small disturbances. This results in a transition to a fundamentally different kind of flow, a turbulent flow.

__________ are largely responsible for the character of a laminar flow in that viscous forces help damp out any perturbations to the flow. Stable laminar flow tends to be associated with low Re. ________, associated with the velocity changes caused by a disturbance have the opposite effect. Inertia forces tend to be destabilizing and thus amplify local disturbances.

A smooth flat plate laminar boundary layer, with no pressure gradient undergoes transition at $Re_e = \ldots$. If disturbances are controlled much higher values of $Re_e$ can result. Similarly, fully developed laminar flow in a tube is observed to be unaffected to large disturbances if the Re based on the hydraulic diameter ≤ 2,300. As shown from viscous stability theory, small disturbances (allowing the use of the linearized theory) present in an external laminar boundary layer with a zero pressure gradient can be stable up to $Re_e = 60,000$. ________ gradients (i.e., decelerating flows) result in lower values of critical Reynolds number, $Re_c$. Favorable pressure gradients (i.e., accelerating flows) enable a higher $Re_c$.

__________ at the surface might be expected to have similar effects, with suction being stabilizing and blowing destabilizing. For an external laminar boundary layer with a zero pressure gradient, the critical $Re_e = 162$.

8.2  The Qualitative Structure of a Turbulent Boundary Layer
Breakdown of a laminar boundary layer at a given x location does not occur uniformly across the boundary layer. Instead, unstable regions referred to as spots develop and burst, spreading laterally as the flow proceeds downstream until the entire boundary layer is engulfed in turbulence. This highly three-dimensional region is called the transition region.

A transition region can be longer than the initial laminar region. However, given a sufficient length of plate, a fully developed turbulent boundary layer would result.

Two regions are observed in a wall bounded turbulent shear layer:

i) a viscous zone adjacent to the wall, where the transfer of heat and momentum can be accounted for by molecular processes, and

ii) a turbulent region that occupies most of the boundary layer further away from the wall. Within this region the velocity is time variant and vortical motions are observed.

The thin viscous region (i.e., the __________) can become locally unstable to flow disturbances and breakdown. This breakdown is referred to as a burst of turbulence. A turbulent burst is highly 3-D and results in an ejection of a large mass of slow moving fluid. As this mass of fluid moves out into the fully turbulent region, a similar mass of higher velocity fluid fills the void at the wall. Within this turbulent region, turbulent kinetic energy (TKE) is generated by the interaction of the __________. While the production of TKE is taking place, ________ interactions are acting to breakdown these eddies into smaller and smaller scales. Once a sufficiently small scale is reached, viscosity acts as the mechanism and transforms the turbulent energy into thermal energy. Under these conditions a new sense of equilibrium is reached in which ________, and a portion of the turbulent energy that is convected and diffused away. Moving downstream the turbulent boundary layer grows, and at a rate greater than a laminar boundary layer. However, the sublayer maintains its same thickness Re and becomes a smaller fraction of the overall boundary layer.
8.3 Wall Coordinates

\[ u^+ = \frac{\bar{u}}{u_t} = \frac{\bar{u}/U_\infty}{\sqrt{c_f/2}} \quad \text{and} \quad u_t = \frac{\mu \Delta \rho/\Delta x}{\rho^{1/2} \gamma_0^{3/2}} \]

\[ y^+ = \frac{u_t y}{v} = \frac{y U_\infty \sqrt{c_f/2}}{v} \]

Within a turbulent boundary layer there is a region where the normalized velocity \(u^+\) is related to the normalized distance from the wall \(y^+\) in the following way and is called the law of the wall:

\[ u^+ = 2.44 \ln y^+ + 5 \quad (8.1) \]

Closer to the wall, within the viscous sublayer,

\[ u^+ = \frac{u_*}{\bar{u}} \quad (8.2) \]

Eqs 8.1 & 2 are in good agreement with the experimental data shown above for two very different values of \(Re_\kappa\). Somewhere below \(y^+ < 50\) the experimental data begins to deviate from the law of the wall. It is at this distance the viscous sublayer appears to become important. For \(y^+ < 5\), eq 8.2 appears valid.

Moving to larger values of \(y^+\) the data is observed to move away from the law of the wall. This region is commonly referred to as the wake region.

The effects of streamwise pressure gradient on a velocity profile, in wall coordinates is presented. The presence of an _________ (i.e., decelerating flow) shows a much larger wake region than the zero pressure gradient case. For a _________ (i.e., accelerating flow) the wake region is diminished and can possibly fall below the law of the wall.

Although the law of the wall provides good agreement with the data, a power law form is often more convenient. A common power law fit that provides a satisfactory correlation out to \(y^+ = 1500\) follows:

\[ u^+ = \]

(8.3)

8.4 An Approximate Solution for a Turbulent Momentum Boundary Layer

Incorporating eq 8.3 into the momentum integral provides a relatively simple algebraic eq for the friction coefficient in a simple turbulent boundary layer.

Although eq 8.3 was developed for \(U_\infty\) constant (i.e., zero pressure gradient) this also turns out to be a very good approximation for strongly accelerated flows, and a fairly good relation for very mild decelerating flows. However, this relation is not good for blowing or suction.

Using eq 8.3 determine \(\tau_w\):

\[ u^+ = \frac{\bar{u}}{u_t} = 8.75 \left[ \frac{y \tau_w / \rho}{v} \right]^{1/3} \]

solving for \(\tau_w @ y = \delta\)

(8.4)

Evaluating the displacement thickness (\(\delta^+\)) and the \(\theta\) using eq 8.3.
Next compute the shape factor, \( H = \frac{\delta^*}{\theta} = 1.29 \)

Expressing eq 8.4 in terms of \( \theta \),

\[
\tau_o = 0.0125 \rho U_{\infty}^2 \left( \frac{\theta U_{\infty}}{v} \right)^{-1/4}
\]  

(8.5)

or using the local skin friction coefficient,

However, the best experimental correlation is that of Schultz-Grunow which is valid for \( Re_{x} > 5 \times 10^5 \):

\[
cr/2 = 0.185 \left( \log_{10} Re_{x} \right)^{-2.584}
\]  

(8.6)

\[
8.5 \text{ An Equilibrium Turbulent Boundary Layer}
\]

An equilibrium turbulent boundary layer is one in which there is a similarity in the outer flow region. Velocity profiles plotted in velocity defect coordinates are universal within this region,

\[
\frac{\bar{u} - U_{\infty}}{u_\tau} = f \left( \frac{y}{\delta^*} \right)
\]

where \( \delta^* = -\int \left[ \frac{\bar{u} - U_{\infty}}{u_\tau} \right] dy \)  

(8.8)

Note: Inner region similarity is not a required condition for the existence of an equilibrium boundary layer.

Velocity profiles within an equilibrium boundary layer, under the condition of adverse pressure gradient and blowing follow:

The following shape factor, proposed by Clauser, was determined to be a constant and independent of \( x \),

\[
G = \int_{\delta^*}^{\infty} \left[ \frac{\bar{u} - U_{\infty}}{u_\tau} \right]^2 d \left( \frac{y}{\delta^*} \right)
\]

(8.9)

Examining the momentum integral for \( \rho = \text{constant} \) provides Defect profiles for an adverse pressure gradient, transpired a measure of the rate of growth of the momentum deficit in turbulent boundary layer

the boundary layer, \( \frac{d(U^2 \theta)}{dx} = \frac{\tau}{\rho} \left[ 1 + B_f + \beta \right] \)  

(8.10)

where \( B_f \), first encountered with laminar boundary layers is a blowing or transpiration parameter, i.e., the ratio of transpired momentum flux to wall shear force,

\[
B_f = \frac{V_o/U_{\infty}}{cr/2} = \frac{\rho V_o U_{\infty}}{\tau_o}
\]

and \( \beta \), the ratio of the axial pressure force to wall shear force is written, \( \beta = \text{or in terms of nondimensional parameters} \), \( \beta = \rho^* H Re_o \left( cr/2 \right)^{1/2} \)

If \( B_f \) or \( \beta \), or both are constant in a turbulent boundary layer, outer region similarity has been observed along with a constant \( G \). The following displays \( G \) as a \( f(B_f + \beta) \)

Recall, similarity was obtained in a laminar boundary layer when \( U_o = cx^n \). Likewise, a turbulent boundary layer under the same velocity variation lead to a constant \( G \), i.e., an equilibrium boundary layer is obtained.

The presence of an equilibrium boundary layer in an accelerating flow occurs if \( K \), the acceleration parameter, is constant along the surface,
\[ K = \frac{\nu}{U_\infty} \frac{dU_\infty}{dx} \]  

(8.10)

Note \( K \) is related to the inner-region pressure gradient parameter \( p^* \);  
\[ p^* = \frac{-K}{(c_t/2)^{3/2}} \]

For incompressible plane flows a momentum integral may be written,  
\[ \frac{1}{U_\infty/\nu} \frac{dRe_\theta}{dx} = \frac{c_t}{2} + \frac{V_\theta}{U_\infty} - K(1 + H)Re_\theta \]  

(8.11)

If \( K \) and \( V_\theta / U_\infty \) are constant w.r.t. \( x \) and if \( K \) is "*+" and if the last two terms sum to a "*" number, then the boundary layer approaches an equilibrium condition for which \( Re_\theta \) is constant, this is referred to as an asymptotic accelerating (or sink) flow. This condition is a special case where a boundary layer has both inner and outer region similarity. Not only is \( Re_\theta \) constant, but so are \( G, H, \beta, \) and \( c_t. \)

For \( K = 0, \) an asymptotic boundary layer will be reached for negative values of \( V_\theta / U_\infty = 0; \) i.e., suction. This type of flow is known as an asymptotic layer. If \( V_\theta / U_\infty = 0; Re_\theta \) at the location where equilibrium occurs depends on \( K, \) large values of \( K \) lead to low \( Re_\theta. \)

\( Re_\theta \) can actually decrease in the flow direction, if \( K \) is large. In fact if \( K \) is sufficiently large, \( Re_\theta \) can be < the critical \( Re_\theta \) necessary for transition from a laminar to turbulent boundary layer. Under this condition turbulent production ceases, the turbulence decays and a laminar boundary layer reemerges. This phenomenon is referred to as _______.

___________ tends to occur when \( K > 3 \times 10^4. \) At much lower values of \( K \) laminar like heat transfer behavior is observed. An example of this is seen in highly accelerated nozzle flows.

### 8.6 The Effect of Surface Roughness

Surface roughness effects occur primarily at the wall. Roughness elements are characterized by \( k_r, \) a characteristic length of an element & the roughness \( Re, \)

\[ Re_k = \]

From experiments three regimes have been observed:

- \( Re_k < 5.0: \) Below which a surface is considered aerodynamically smooth.
- \( 5.0 < Re_k < 70.0: \) Transitional roughness, where both smooth and rough wall characteristics exist.
- \( Re_k > 70.0: \) A fully rough surface exists. In this region \( c_t \) becomes independent of \( Re, \) i.e., viscosity is no longer a significant variable.

\( Re_k > 70.0, \) the viscous sublayer disappears and shear stress is produced by a mechanism other than viscous shear. This mechanism is pressure drag. For \( y' > Re_k, \) experimental data in the rough region for zero pressure gradient and \( V_\theta = 0 \) suggest,

\[ u^+ = 2.44 \ln y^+ + 2.44 \ln \left( \frac{32.6}{Re_k} \right) \]  

(8.12)

\& for a rough surface boundary layer with \( \theta / \delta = 0.097\)

\[ c_t / 2 = \frac{0.168}{\left( \ln(8649 / k_r) \right)^2} \]  

(8.13)

![FIGURE 11.14](image)

Comparison of Eq. (8.13), and the mixing-length model, with experimental data of Pimentel for a rough surface composed of packed balls at constant free-stream velocity (IDENT 71374 fully rough surface, 1.27 mm spheres, \( k_r = 0.787 \) mm.).
8.7 The Transpired Turbulent Layer
A porous wall can either inject or remove fluid from the external boundary layer and thus, \( V_x \neq 0 \). Thus a transpired boundary layer is one where blowing, suction, or mass transfer takes place at the surface.

Recall, \( B_i \) equaling a nonzero constant enabled similarity solutions to be obtained for a laminar boundary layer. Likewise, \( B_i = \) constant enables the existence of an equilibrium boundary layer.

Considerable alteration of the boundary layer structure occurs when transpiration is applied. It affects both the \( \tau \) distribution and sublayer thickness.

Two velocity profiles are presented for the case of a turbulent boundary layer with strong blowing and strong suction; where, \( F = \)

Note:
1- Under blowing the wake can be very large and resembles the wake for an adverse pressure gradient (see Fig 11-4).
2- Suction has the opposite effect & resembles that of a favorable pressure gradient. Under strong suction wake can completely disappear.

8.8 The Effect of Freestream Turbulence

\[
TI_u = \frac{U}{U_{\infty}}
\]  

(8.14)

where the numerator is the RMS of the streamwise velocity fluctuation.

The main effect of freestream turbulence has been found to occur in the outer part of the boundary layer, the wake region, where the turbulence increases the mixing.

The sublayer and inner part of the logarithmic region are in general unaffected, i.e., the influence of \( TI_u \) is ___________ as the wall is approached.

The following displays a wake being depressed as \( TI_u \) increases.

The value of \( u^+ \) at the outer edge of a turbulent boundary layer equals \( 1/\sqrt{c_f/2} \) thereby showing that increased \( TI_u \) increases \( c_f \). At present there is no general correlation between \( TI_u \) and \( c_f \).

**FIGURE 11-13**
The effect of transpiration on velocity profiles, constant free-stream velocity Andersen\(^b\) and Simpson\(^b\).

**FIGURE 11-17**
The effect of free-stream turbulence on velocity profiles; \( Re_\theta \) in parentheses (data of Johnson and Johnston\(^b\)).
Velocity profiles for a turbulent flow over a smooth flat plate with a zero pressure gradient are shown.

A typical logarithmic profile in wall coordinates is observed to exist between \( y^+ = 50 \) to \( y^+ = 1000 \).

In a similar fashion a logarithmic region can be observed in defect coordinates.

The following mean velocity profiles were experimentally obtained and are presented to demonstrate the effect of pressure gradient on the turbulent boundary layer.

**FIGURE 6-4**
Experimental turbulent-boundary-layer velocity profiles for various pressure gradients.