Part II

Probability, Design and Management in NDE
Probability Distributions

The probability that a flaw is between $x$ and $x + dx$ is $p(x)dx$

$x$ is the flaw size

$p(x)$ is the probability density

\[
\int_{0}^{\infty} p(x)dx = 1
\]

$x_a = \int_{0}^{\infty} x p(x)dx$

$x_a$ average flaw size

The probability that a flaw is smaller than $x$ is $P(x)$

\[
P(x) = \int_{0}^{x} p(x)dx
\]

$P(x)$ cumulative probability

\[
\lim_{x \to \infty} P(x) = 1
\]

$P(x_m) = 0.5$

$x_m$ median flaw size
**Probability of Detection**

The probability that a flaw of given size $x$ is detected:

$$POD(x)$$

$$\frac{\partial POD}{\partial x} \geq 0$$

$$\lim_{x \to \infty} POD(x) = 1$$

The probability of detecting a flaw of size between $x$ and $x + dx$ is

$$POD(x) p(x) dx$$

The probability of having an undetected flaw of size between $x$ and $x + dx$ is

$$[1 - POD(x)] p(x) dx$$

After NDT, the modified probability density of the reduced ensemble

$$p'(x) = k[1 - POD(x)] p(x)$$

Re-normalization

$$\int_{0}^{\infty} p'(x) dx = 1$$
Probability of Detection

- Operator training, alertness and confidence
- Correct application of the proper technique
- Environment of the test (laboratory, field)
- Material homogeneity and isotropy
- Flaw characteristics
- Shape of the part, surface roughness
- Calibration and capability of the system
- etc.

(US Air Force maintenance facilities, round-robin inspection of box-wing)
Probability Distributions

Flaw Size, $x$

Probability Density, $p(x)$

before NDT

after NDT

Cumulative Probability, $P(x)$

before NDT

after NDT

POD

Flaw Size, $x$
Noise-Limited POD

grain noise in polycrystalline materials

![Diagram of grain structure](image)

![Graph showing signal and noise](image)

![Graph showing probability density](image)
Misqualified Cases

\[ N = N_D + N_{DF} \]

\[ N_{MQ} = N_{MF} + N_{FA} \]

\( N \) total number of components

\( N_D \) defective components

\( N_{DF} \) defect-free components

\( N_{MQ} \) misqualified components

\( N_{MF} \) missed flaws

\( N_{FA} \) false alarms

\[ N_{MF} \approx N_D \int_0^{V_D} p_F(V) \, dV \]

\[ N_{FA} \approx N_{DF} \int_{V_D}^{\infty} p_N(V) \, dV \]

\( p_F \) probability density of the flaw signal

\( p_N \) probability density of the material noise

\( V \) amplitude

\( V_D \) detection threshold
Modified Probabilities after NDE

\( x \) flaw size

\( p(x) \) original probability density of flaw size

\( POD(x) \) probability of detection

among “defective” components

\[
p_{dd}(x) = k_d \cdot POD(x) \cdot p(x)
\]

\( p_{dd}(x) \) probability density of flaw size among “defective” components

\( k_d \) re-normalization factor (to assure that \( \int_{0}^{\infty} p_{dd}(x) \, dx = 1 \))

among “defect-free” components

\[
p_{df}(x) = k_f \cdot [1 - POD(x)] \cdot p(x)
\]

\( p_{df}(x) \) probability density of flaw size among “defect-free” components

\( k_f \) re-normalization factor (to assure that \( \int_{0}^{\infty} p_{df}(x) \, dx = 1 \))

**Sensitivity**

\( V = S(x,...) \)

\( S \) sensitivity function (typically nonlinear, multi-variable)

for small defects \( V \propto x^n \), where \( n \) could be as high as 3-5

for large defects \( V \approx \) constant
Failure Prevention

- Load environment
- Fracture properties
- NDT

Probability Density

- Stress demand
- Service degradation
- Strength available

- Nominal load
- Peak load
- Nominal strength

Safety factor
The probability that a specimen fails between $t$ and $t + dt$ is $f(t) dt$

$$f(t)$$ is the failure rate (probability density)

The probability that a specimen fails after a given time $t$ is $P(t)$

$$P(t + dt) - P(t) = [1 - P(t)] f(t) dt$$

$P(t)$ is the probability of failure

$$P(t + dt) - P(t) = [1 - P(t)] f(t) dt$$

Initial failure ($P(t) << 1$)

$$\frac{dP(t)}{dt} \approx f(t)$$

Constant failure rate

$$\frac{dP(t)}{dt} = [1 - P(t)] f_0$$

$$P(t) = 1 - e^{-f_0 t}$$

Ultimate probability of failure

$$\lim_{t \to \infty} P(t) = 1$$
Failure Rate and Probability of Failure

Probability of failure vs. time (1,000 hours)

- Probability of failure increases over time.
- Failure rate (per 1,000 hours) remains constant.

Graphs illustrate the relationship between time and probability of failure, showing how the likelihood of failure grows as time progresses.
Material fatigue means changes in properties that can occur due to repeated application of stresses/strains and usually means a process which leads to cracking or failure.

- **mechanical fatigue**
- thermo-mechanical fatigue
- creep-fatigue
- corrosion-fatigue, etc.

A common feature of these failure processes is that they take place under the influence of loads whose peak values are considerably lower than the “safe” loads estimated on the basis of static fracture analysis.
Fatigue Life

I. **Crack Initiation** extends from the commencement of service until the moment when a terminal fatigue crack appears
   a) before crack nucleation
   b) after crack nucleation

II. **Crack Growth** starts with the moment of crack initiation and lasts until the abrupt *failure* in the remaining cross-sectional area

The duration of crack initiation and crack propagation depend on:

- material properties (strength, toughness, etc.)
- mechanical load (type, level, load ratio, etc.)
- environmental effects (temperature, humidity, etc.)
- surface conditions (roughness, contamination, corrosion, etc.)
Stages of Fatigue and the State of NDE

- Substructural and Microstructural Damage
- Microscopic Crack Formation
- Growth and Coalescence into Dominant Cracks
- Stable Propagation of Dominant Macrocrack
- Structural Instability or Fracture

- Crack Nucleation
- Crack Initiation

- Field Inspection Techniques
- State-of-the-Art Inspection Techniques
- Experimentary Laboratory Techniques
- Interdisciplinary Research Techniques
Fracture Mechanics

Stress singularity

\[
\lim_{x' \to 0} \sigma(x') \propto \frac{K}{\sqrt{x'}}
\]

Stress intensity factor

\[
K = \sigma_a \sqrt{\pi a}
\]

\(a\) is the half-width of the crack (2D slit or Griffith crack)
Critical Crack Size

Critical stress intensity factor, fracture toughness

\[ K_c \]

Critical crack size

\[ 2a_c = \frac{2 K_c^2}{\pi \sigma^2} \]

Crack growth rate (Paris Law)

\[ \frac{da}{dn} = A \Delta K^m = B \left( \frac{\Delta K}{K_c} \right)^m \]

\( n \) number of cycles

\[ \Delta K = K_{\text{max}} - K_{\text{min}} \] stress intensity factor range

\( A, B, m \) material (temperature \( T \), stress ratio \( R = \sigma_{\text{min}}/\sigma_{\text{max}} \), etc.) variables

\[ B = AK_c^m \]
Influence of Stress Ratio

Paris Law

\[
\frac{da}{dn} = A \Delta K^m = A(K_{\text{max}} - K_{\text{min}})^m
\]

\[K_{\text{max}} = \sigma_d^{\text{max}} \sqrt{\pi a}\]  and  \[K_{\text{min}} = \sigma_d^{\text{min}} \sqrt{\pi a}\]

average stress

![Graph showing average stress over time with different stress ratios R = 0, R = 0.3, R = 0.6, and R = -1.]

Paris-Walker Law

\[
\frac{da}{dn} = A \frac{(K_{\text{max}} - K_{\text{min}})^m}{(1 - R)^w}
\]

\[R = \frac{\sigma_d^{\text{min}}}{\sigma_d^{\text{max}}}\]
Design Philosophies

Engine Structural Integrity Program (ENSIP, USAF)

Retirement for Age

Low-Cycle-Fatigue Design Criteria (safe life)

- based on statistical lower bound
- 1 in 1,000 components is predicted to initiate a 1/32” crack

![Diagram showing typical distribution of usage and life with mean, lower bound, and -3σ markers.](image)
This photo emphasizes that the USAF retires a great number of turbine engine disks. Disks represent a significant asset, thus discarding them prior to their full useful life represents a significant cost to the Air Force. Technologies are required to more fully use the fatigue lives inherent in turbine engine disks.
Retirement for Cause

Damage-Tolerant Design Criteria

1-3 safety inspections during service life

Crack Size vs. Cycles (or equivalent)

Crack nucleation → crack initiation → crack limit → terminal crack

1st inspection → 2nd inspection → 3rd inspection → retirement