Iterative inversion method for eddy current profiling of near-surface residual stress in surface-treated metals

Bassam A. Abu-Nabah, Peter B. Nagy*

Department of Aerospace Engineering and Engineering Mechanics, University of Cincinnati, Cincinnati, OH 45221-0070, USA

Received 12 April 2006; accepted 1 May 2006
Available online 16 June 2006

Abstract

Because of their frequency-dependent penetration depth, eddy current measurements are capable of mapping the near-surface depth profile of the electric conductivity. This technique can be used to nondestructively characterize the subsurface residual stress distribution in certain types of shot-peened metals, e.g., in nickel-base superalloys. In this paper, a highly convergent iterative inversion procedure is presented to predict the frequency-independent intrinsic electric conductivity depth profile from the frequency-dependent apparent eddy current conductivity (AECC) spectrum. The proposed technique exploits three specific features of the subsurface electric conductivity variation caused by near-surface residual stresses in shot-peened metals. First, compressive residual stresses are limited to a shallow surface region of depth much less than typical probe coil diameters. Second, the change in electric conductivity due to residual stresses is always very small, typically less than 1%. Third, the electric conductivity depth profile is continuous and fairly smooth. The accuracy of the proposed iterative inversion procedure is one order of magnitude better than that of the previously developed simpler method (J Appl Phys 2004;96:1257).

Keywords: Eddy current; Inversion; Shot peening; Residual stress

1. Introduction

Surface enhancement methodologies, such as shot peening (SP), low-plasticity burnishing (LPB), and laser shock peening (LSP), are widely used to improve the fatigue resistance of fracture-critical structural components without weight or dimensional increase. In SP, the surface of the component is hit repeatedly with a large number of small steel, glass, or ceramic shot, making overlapping indentations on the surface. The impacts cause plastic deformation within a depth of about 50–200 μm from the surface and impart compressive residual stresses, thus improving the fatigue life of the component. In addition, the surface is slightly hardened and strengthened by cold-working. This process is extensively used on critical components of irregular shapes, especially in areas that may be subject to stress concentration, parts subjected to cyclic loading, or parts existing in a corrosive or erosive environment, such as shafts, gears, springs, oil-well drilling equipment, and jet-engine parts like turbines and compressor blades. Since fatigue failure mainly results from tensile stresses, having the surface in compression offsets any tendency toward such failure. Unfortunately, SP is affected by numerous process variables, which results in manufacturing variations and makes it advantageous to use some kind of nondestructive inspection to provide quality control after peening. Furthermore, the relatively high level of cold-work caused by SP, sometimes in excess of 30–40% plastic strain, tends to reduce the thermomechanical stability of the protective compressive layer and leads to accelerated residual stress relaxation at elevated operating temperatures. Therefore, repeated testing of the relaxed residual stress profile during scheduled maintenance is required to reliably assess the remaining fatigue life of the component.

Currently, the best available technique to characterize the near-surface residual stress profile in surface-treated components is based on X-ray diffraction (XRD), which measures the 3 elastic strain in the crystal lattice so that the
prevailing stress can be calculated assuming linear elasticity. A significant drawback of XRD is that, at typical intensity levels and inspection wavelengths, it offers only rather limited penetration depth of about $5-20\,\mu m$, which is an order of magnitude lower than the penetration depth of the compressive residual stresses produced by most surface enhancement methods. Therefore, to cover at least the crucial compressive part of the whole residual stress profile by XRD, successive destructive layer removal is required.

Due to the inherently destructive nature of XRD, novel nondestructive evaluation (NDE) methods are needed to assess the complete near-surface residual stress profile in surface-treated components.

Due to its frequency-dependent penetration depth, eddy current conductivity inspection is considered one of the most promising NDE methods for near-surface residual stress assessment. Mounting evidence indicates that the characteristic dependence of the electric conductivity on stress, i.e., the so-called piezoresistivity effect, can be exploited for residual stress measurements in certain surface-treated metals profiling [1–11]. Unfortunately, the measured apparent eddy current conductivity (AECC) depends not only on the sought residual stress, but it is also affected by cold-work and surface roughness, as it is shown schematically in Fig. 1. In most metals, the stress-related electric conductivity variation is quite weak and rather difficult to separate from parallel electric conductivity and, in certain cases, magnetic permeability variations caused by cold-work. In addition, surface roughness can also cause a perceivable loss of AECC at high inspection frequencies when the eddy current has to follow a tortuous path to stay within the shallow electromagnetic skin depth of the material [12–14].

In some paramagnetic materials, such as aluminum and titanium alloys, SP actually decreases the AECC by $\approx 1-2\%$, which indicates that cold-work and surface roughness effects dominate the observed change [6–10]. In contrast, shot-peened nickel-base superalloys exhibit an increase in the electric conductivity parallel to the surface at increasing inspection frequencies and peening intensities [10]. Results indicate that in these materials the observed increase in AECC is dominated by compressive residual stresses, whereas surface roughness, increased dislocation density, and increased magnetic susceptibility are all known to decrease the measured AECC. Since nickel-base superalloys crystallize dominantly in cubic symmetry, crystallographic texture caused by cold-work below the surface is not expected to affect the electric conductivity at all, but subtle microstructural variations can still produce a significant change in AECC. On partially relaxed shot-peened Waspaloy specimens, it was found that the measured AECC is roughly proportional to the remaining residual stress. Furthermore, when shot-peened Waspaloy specimens were fully relaxed with the intention of

![Fig. 1. A schematic representation of physics-based eddy current residual stress profiling in surface-treated components. The highlighted parts correspond to the forward and inverse problems addressed in this paper.](image-url)
completely eliminating the remaining residual stresses in the sub-surface layer, the excess AECC also completely disappeared even though the cold-work did not entirely vanish. These observations support the conclusion that AECC is more sensitive to residual stress than to cold-work, although it has been found recently that an overestimation in the absolute magnitude of the residual stress could occur close to the surface if cold-work effects are left uncorrected [15].

Fig. 1 summarizes the complex relationships that exist between the three main effects of SP on one side and the measured AECC spectrum on the other side. Experimental evidence suggests that in the studied nickel-base superalloys (Waspaloy, IN100, and IN718) neither residual stress nor cold-work causes any perceivable magnetic change and, at least up to about 20–30 MHz, surface roughness effects are also negligible [10,15]. Under these conditions, in order to find the sought near-surface residual stress profile $\tau(z)$ from the measured AECC spectrum $G(f)$, a two-step inversion procedure must be implemented. First, the electric conductivity profile $\sigma(z)$ must be calculated from $G(f)$, which is the main topic of the present paper. Second, $\tau(z)$ must be calculated from $\sigma(z)$. It should be mentioned that in the case of high peening intensities (above Almen 6A) this second step usually requires compensation for coldwork effects in order to avoid overestimation of the residual stress magnitude.

In the following an improved iterative technique is proposed for inverting the measured AECC spectrum $G(f)$ in terms of the electric conductivity depth profile $\sigma(z)$ based on a previously introduced simplistic inversion technique [16]. The improved inversion procedure will be validated by comparison to numerical simulations. It will be shown that the proposed iterative inversion procedure is robust, highly convergent, and its inversion accuracy is roughly one order of magnitude better than that of the previously developed simpler method.

### 2. Forward eddy current problem

Let us consider eddy current characterization of a nonmagnetic half-space of continuously varying near-surface conductivity distribution that depends only on the depth below the surface. Then, the electric conductivity distribution can be written as $\sigma(z) = \sigma_p(z)$; where $\sigma$ is the electric conductivity of the intact material deep below the surface and $\rho(z)$ is a depth profile that approaches unity at large depths. The previously mentioned AECC of such an inhomogeneous medium at a given inspection frequency is defined as the electric conductivity of an equivalent homogeneous medium that would produce the same complex electric coil impedance as the inhomogeneous medium [10]. The lift-off distance of the equivalent homogeneous medium, which is needed to achieve the identical coil impedances, is called the apparent eddy current lift-off (AECL) of the inhomogeneous medium. This second parameter is less useful for the purposes of characterizing the inhomogeneous specimen and, in practice, it is highly susceptible to experimental variations, therefore it is usually discarded as an irrelevant byproduct of the matching process.

The forward eddy current problem usually requires two separate steps. First, the frequency-dependent complex electric impedance of the probe coil must be determined for a given coil geometry and lift-off distance. Second, the frequency-dependent AECC is calculated from the complex impedance of the probe coil by finding the electric conductivity of the equivalent homogeneous medium. In analytical studies, the first step is usually based on some variation of the analytical model originally developed by Dodd and Deeds for a homogenous half-space covered with a single conductive layer [17], which was later extended to arbitrary number of layers using the transfer matrix method [18]. Based on this approach, several different numerical and, for certain profiles, also analytical solutions are available in the literature [19–26].

In the second step of the forward problem, the frequency-dependent AECC is determined from the actually measured or calculated complex electric coil impedance. After bracketing the conductivity range of interest with two homogeneous reference blocks, the complex electric coil impedance is measured or calculated without a lift-off ($l = 0$) and with a lift-off ($l = s$). Then, the AECC and AECL parameters of the inhomogeneous medium are estimated, most often using a simple four-point linear interpolation procedure. This second step effectively separates the material effects associated with the specimen from size and shape effects associated with the probe coil [10,16]. Therefore, the AECC spectrum $G(f)$ of the specimen is more suitable for materials characterization purposes than the directly measured or calculated complex electric impedance $Z(f)$ of the probe coil, which is sensitive to both.

Recently, in order to directly calculate the frequency-dependent AECC spectrum from the depth-dependent conductivity profile of inhomogeneous conductors and vice versa, a set of 7 simple physics-based analytical approximations were proposed by Yu and Nagy [16]. Two specific features of the electric conductivity variation observed in shot-peened metals facilitated the adaptation of simple analytical approximations for both the forward and inverse electromagnetic problems. First, compressive residual stresses are limited to a very shallow depth, which is much less than typical probe coil diameters, and the penetration depth of coldwork effects is even less. In order to characterize such conductivity profiles, the inspection frequency has to be chosen so that the standard eddy current penetration depth $\delta$ in the conductor is comparable to the depth of the conductivity variation (50–200 μm), which is much less than the diameter of typical probe coils (2-5 mm). Second, the change in electric conductivity due to residual stresses is always very small, typically less than 1%.
Exploiting these features, a simple 1D approximation was developed for the forward problem based on the electromagnetic plane wave reflection coefficient $R(f)$ of the inhomogeneous conductor, which assumes that the probe coil is infinitely large [16]. In comparison with numerical results obtained with a commercially available electromagnetic simulation software, the proposed simple 1D approximation was validated when the probe radius was sufficiently larger than the thickness of the affected surface layer. It is important to point out that this approximate analytical technique does not require having two calibration blocks of known conductivity to determine the frequency-dependent AECC since the effects of probe size and shape are inherently neglected. Under certain conditions, e.g., in the case of apparent conductivity increase due to near-surface compressive residual stresses [10,16] or apparent conductivity loss due to surface roughness [13,14], it becomes advantageous to evaluate the AECC as a function of frequency instead of just the complex coil impedance.

In the case of a known 1D conductivity profile, the electromagnetic surface impedance $Z(0)$ of the inhomogeneous half-space can be determined by repeated application of the so-called wave-guide equation with sufficiently small discretization steps starting deep enough below the surface, where the electric conductivity approaches the intact material using the well-known standard penetration depth of the eddy current distribution and the integrating effect of this simplification, the exponential decay of the electromagnetic plane wave reflection coefficient $R(f)$ of the conductor can be directly calculated as follows:

$$R(f) = \frac{\eta_0 - Z(0)}{\eta_0 + Z(0)},$$

where $\eta_0 = \sqrt{\mu_0/\varepsilon_0}(\approx 377 \Omega)$ is the intrinsic electric impedance of free space and $\mu_0$ and $\varepsilon_0$ are the magnetic permeability and electric permittivity of a vacuum, respectively. Then, the sought near-surface electric conductivity profile can then be evaluated from [16]

$$\Gamma(\delta) \approx \frac{4\pi f \mu_0 \varepsilon_0}{\mu_0 [1 - \text{Re}\{R(f)\}]^2},$$

where $\mu_0$ is the magnetic permeability of the intact conductor (typically, $\mu_0 \approx \mu_0^0$). Comparison to the AECC calculated numerically using the commercially available Vic-3D software, which uses the volume integral method, showed that the 1D incremental layer approximation for the forward electromagnetic problem can very accurately predict the AECC from known depth-dependent conductivity profiles in a nonmagnetic half-space. Due to the obvious advantages of the direct 1D approximation over the conventional indirect method of first calculating the complex coil impedance of a finite-size coil and then evaluating this impedance for the AECC using reference calculations on homogeneous calibration blocks of known conductivity, in the present work the 1D incremental layer approximation method will be used exclusively to solve the forward problem.

### 3. Inverse eddy current problem

The sought near-surface electric conductivity profile can be assessed either directly from the measured complex coil impedance $Z(f)$ or from the AECC spectrum $\Gamma(\delta)$ after first removing the effects of coil size and geometry. For recent reviews on the eddy current inversion problem see Refs. [27–29]. In spite the easier physical interpretation of the frequency-dependent AECC spectrum versus the more convoluted electric coil impedance, essentially all the available studies in the literature are based on the impedance method [19–35].

#### 3.1. Simplistic inversion model

Recently, a very simple approximation was proposed for eddy current characterization of near-surface conductivity variations in surface-treated metals based on direct inversion of the AECC spectrum [16]. This approximation exploits that the electric conductivity variation $(i)$ is limited to a shallow surface layer, which is much smaller than the probe coil diameter, $(ii)$ varies smoothly, and $(iii)$ is relatively weak ($\approx 1\%$). Based on these assumptions, the inverse problem was linearized using the Born approximation, i.e., by replacing the field in the inhomogeneous conductor with the field that would exist in a homogeneous conducting half-space [32]. In this approximation, the standard penetration depth of the eddy current distribution, which equals the actual penetration depth when finite-size effects are neglected, can be calculated from the conductivity of the intact material using the well-known formula $\delta_i(f) = 1/\sqrt{\pi f \mu_0 \sigma_i}$. Furthermore, according to the simplistic approximation, the exponential decay of the eddy current distribution and the integrating effect of this distribution below the surface were also neglected and it was crudely assumed that the measured AECC corresponds to the actual electric conductivity at a given depth $z(f)$ below the surface

$$z = \delta_i(f)/w,$$

where $w$ is a scaling factor that was arbitrarily chosen to be equal to two in the earlier study of Yu and Nagy [16] but could take any other value within reasonable limits.

The schematic diagram of the numerical procedure used to validate this simplistic inversion model is shown in Fig. 2(a). The starting point is an adjustable electric conductivity profile $\sigma(z)$, which we will refer to as exact in the following. This profile is chosen so that it represents a typical residual stress distribution measured by XRD in shot-peened nickel-base superalloy specimens and assuming average piezoresistivity coefficients for the material. The AECC $\Gamma(\delta)$ will be simulated using the 1D forward approximation whose excellent accuracy was numerically verified earlier [16]. We will refer to this $\Gamma(\delta)$ as the input AECC (in the case of experimental adaptation of the inversion procedure this is the measured AECC spectrum). According to the simplistic model, the inverted
conductivity profile $\sigma_0(z)$ can be directly calculated from the input AECC spectrum using

$$\sigma_0(z) = \delta (f)/w = \Gamma(f),$$

where the subscript 0 refers to the fact that no iteration was used to increase the accuracy of the inversion. In the simplistic model there is no compensation for inversion errors other than choosing the value of $w = 2$ so that the best overall agreement could be achieved between the exact conductivity profile $\sigma(z)$ and the inverted profile $\sigma_0(z)$. In the case of typical conductivity profiles of shot-peened metals, numerical comparison to exact solutions indicated that this simplistic approximation offers an acceptable agreement over most of the depth profile but breaks down both close to the surface and at large depths. In the next section we propose to eliminate, or at least greatly reduce, these inversion errors by exploiting the fact that the simple and straightforward 1D plane wave approximation yields very accurate results for the forward eddy current problem whenever the conductivity variation is limited to a depth negligible relative to the diameter of the probe coil therefore it can be used in an iterative scheme to reduce the otherwise significant inversion errors of the simplistic model.

3.2. Iterative inversion model

The main goal of this paper is to illustrate that much more accurate inversion can be achieved by iterative application of the same principle in a feed-back loop that relies on the outstanding accuracy and speed of the 1D forward approximation, as it is shown schematically in Fig. 2(b). For any given value of $w$, the frequency and depth coordinates can be directly transformed from one to the other using Eq. (3). The proposed iterative algorithm is defined by

$$\sigma_{n+1}(z) = \sigma_n(z) + \Gamma(z) - \Gamma_n(z),$$

where $n = 0, 1, 2, \ldots$ is the order of iteration, $\Gamma(f)$ is the input AECC as before, and $\sigma_n(z)$ and $\Gamma_n(z)$ are the calculated electric conductivity depth profile and AECC spectrum in the $n$th iteration, respectively.

In order to demonstrate the proposed iterative inversion procedure via numerical simulation, the input AECC $\Gamma(f)$ has to be calculated first for a known exact near-surface depth dependent conductivity profile $\sigma(z)$. Like in the simplistic model reviewed in the previous section, the frequency and depth coordinates can be directly transformed from one to the other using Eq. (3), though the value of $w$ is left open to optimize the accuracy and convergence speed of the iterative procedure. In the 0th-iteration $\Gamma_0(f) = \Gamma(f)$ is chosen, which is then transformed to get $\Gamma_0(z)$. The first-iteration depth-dependent conductivity profile $\sigma_1(z)$ is then calculated using Eq. (5). After each repeated application of Eq. (5), the $n$th-order AECC spectrum $\Gamma_n(f)$ must be first calculated from the $n$th-order conductivity profile $\sigma_n(z)$ to make the next iteration to get $\sigma_{n+1}(z)$, as it is illustrated in Fig. 2(b). It will be shown in the next section that the result of this iterative procedure quickly converges to the actual electric conductivity distribution, i.e.,

$$\lim_{n \to \infty} \sigma_n(z) = \sigma(z).$$

Although the value of $w$ will be shown not to be critical in a fairly wide range, it will be illustrated that the best ultimate accuracy and the fastest convergence can be achieved when this parameter is slightly less than 2, which was the arbitrarily chosen scaling factor in the simplistic model.

3.3. Accuracy and convergence of the iterative inversion model

The improved accuracy and fast convergence of the proposed iterative procedure will be demonstrated in this section using numerical simulations. We used for this purpose the commercially available Vic-3D software that is particularly well suited for simulating the axisymmetric electromagnetic system consisting of a test specimen and a probe coil in air [31]. The nominal conductivity of the intact material deep below the surface is $\sigma_1 = 8.7 \times 10^5 \text{ S/m}$, which is equivalent to $\sigma_1 = 1.5\%$ IACS, and the magnetic permeability is $\mu = \mu_0 = 4\pi \times 10^{-7} \text{ V s/A m}$. For illustration purposes, we chose the input conductivity profile shown in Fig. 3(a), which is representative of shot-peened nickel-base superalloys. It has been shown recently that, in addition to the residual stress effect, the presence of cold-work in
nickel-base superalloys cause a substantial increase in the measured AECC [15]. For the purposes of this paper, the selected depth-dependent conductivity profile can be considered as the combined effect of near-surface residual stress and cold-work in shot-peened nickel-base superalloys. The additional surface roughness induced loss of the AECC was not considered in the present study.

The electric conductivity variation is limited to a shallow depth below the surface and the relative change goes to zero at approximately $z = 5\,\text{mm}$. Experimental evidence shows that the maximum relative conductivity change due to SP in nickel-base superalloys is typically less than 1%. In this analysis, the maximum relative conductivity change was $\approx 0.22\%$, which represents a moderately peened (Almen 4-6 A) component after partial relaxation. Based on these features of the near-surface conductivity profile, the simple 1D approximation [16] is sufficient for estimating the AECC as a function of frequency in the present study. The 1D forward approximation must be started sufficiently deep below the surface, at around $z = 5\,\text{mm}$ for the selected profile shown in Fig. 3(a), where the relative change in conductivity is almost zero. From that point, the electromagnetic impedance is calculated repeatedly from layer to layer with sufficiently small incremental steps toward the surface. Finally, the plane-wave reflection coefficient $R(f)$ is calculated from the surface impedance by applying Eq. (1), which then can be used to estimate the input AECC spectrum using Eq. (2), which is shown in Fig. 3(b).

As mentioned earlier in this paper, the proposed iterative inversion technique is a modification of the simplistic inversion technique [16], which assumed that the eddy current distribution is localized at half of the standard penetration depth ($w = 2$), so that at any given frequency, the AECC corresponds directly to the actual electric conductivity at that depth. Fig. 4(a) shows a comparison between the exact conductivity profile $\sigma(z)$ and the inverted conductivity profiles $\sigma_0(z)$ calculated from the input AECC spectrum $\Gamma(f)$ without iteration ($n = 0$) using different values of the scaling factor $w$. It is clear that, without iteration, $w = 2$ yields the best overall fit between the exact and inverted conductivity profiles. This conclusion is also

![Fig. 3](image-url)  
![Fig. 4](image-url)
confirmed in Fig. 4(b) which shows a comparison between the input AECC spectrum $I(f)$, which serves as the input data for the 0th iteration, and the reconstructed AECC spectrum $I_0(f)$ (which would serve as the input data for the next iteration if the process were continued). These results verify that the value of $w = 2$, which was arbitrarily chosen in the simplistic model without detailed error analysis, indeed represents the best choice for the scaling factor if no iteration is used. Therefore, in the following test $w = 2$ will be used as our first guess to demonstrate the convergence of the proposed iterative inversion procedure.

Continuing the iterative procedure as it is schematically shown in Fig. 2(b) and algebraically defined in Eq. (5), the discrepancy between the exact conductivity profile $\sigma(z)$ and the inverted conductivity profile $\sigma_0(z)$ quickly diminishes. Fig. 5(a) illustrates that the discrepancy between the 0th-iteration conductivity profile $\sigma_0(z)$ and the exact conductivity profile $\sigma(z)$ has been reduced to almost half in just one additional step, i.e., in the first-iteration profile $\sigma_1(z)$, and further iteration quickly eliminates most of the remaining discrepancy in just a few steps. Furthermore, as it is illustrated in Fig. 5(b), solving the forward problem on the inverted conductivity profiles leads to an increasingly better reconstruction of the input AECC $I(f)$ that was initially obtained form the exact conductivity profile $\sigma(z)$.

One inherent advantage of the iterative approach over the simplistic model is that it is much less sensitive to the value of the arbitrarily chosen scaling factor $w$. To verify this assumption, which is an indication of the robustness of the iterative procedure, different $w$ values have been tested. Repeated application of Eq. (5) up to $n = 10$ showed that the exact conductivity profile can be reconstructed using a wide range of scaling factors, especially at large depths, but discrepancies caused by numerical instabilities start to show up at low depths when the scaling factor $w$ exceeds 2, as it is shown in Fig. 6. Since the discrepancy at large depths is less significant from the point of view of residual stress assessment and life prediction, it is important to find...
the best scaling factor that will reduce the discrepancies throughout most of the conductivity profile without causing numerical instabilities at low depths.

In order to study the convergence rate of the proposed iterative inversion technique under different scaling factors, we ran the process up to \( n = 20 \) iterations. Fig. 7 shows that the best convergence is reached between \( w = 1.5 \) and 2, where the first five iterations will be sufficient to achieve a reasonable convergence toward the exact conductivity profile. Fig. 7(a) shows the averaged absolute value of the difference between the inverted and exact conductivity profiles after increasing numbers of iteration using a certain scaling factor \( w \). On the other hand, Fig. 7(b) shows the corresponding maximum difference between the inverted and exact conductivity profiles, which is a better indication whether the inverted conductivity profile diverges from the exact profile at low depths, which happens especially at scaling factors higher than \( w = 2.5 \) where the system becomes numerically unstable as shown in both Figs. 6(a) and 7. Moreover, from looking at the results in Fig. 7, it can be concluded that the highest convergence rate is achieved in the very first iteration and that the second iteration is sufficient to achieve reasonable inversion accuracy. Further iteration improves the accuracy very little, but might lead to numerical instabilities. Actually, the difference between the conductivity profiles produced in the second and fifth iterations is negligible with respect to the exact conductivity profile. The optimum scaling factor for the proposed iterative inversion technique seems to be around \( w \approx 1.7 \), which showed the best convergence starting from the second iteration. Since, the differences between the inverted and exact conductivity profiles are very small as long as the scaling factor remains in the range between \( w = 1.5 \) and 2, in our further investigations we will keep using \( w = 2 \) to remain consistent with the previous scaling factor introduced in the simplistic inversion technique [16].

3.4. Robustness of the iterative inversion model

Until this point, the input AECC spectrum was assumed to be exact within the limitations of the 1D approximation as it was directly calculated from the exact conductivity profile. In order to more realistically simulate what happens in actual AECC measurements, a random noise of \( \pm 0.01\% \) normalized AECC variation was superimposed on the input AECC throughout the entire spectrum as shown in Fig. 8(b). Although the relative error is very small, the useful signal itself has a peak value of 0.22\% only, therefore the signal-to-noise ratio (SNR) is only about 20 (26 dB). This type of random noise can be usually eliminated by using high-precision eddy current instruments and extensive point averaging at each inspection frequency. It was found that even such a relatively small random error can lead to numerical instabilities of the iterative inversion in the very first iteration. The reason for this lack of stability is that neglecting the smoothening effect of the continuous eddy current density distribution forces the iterative procedure to create unrealistically high, essentially random variations in the inverted conductivity profile to reproduce the random variations observed in the input AECC spectrum.

As we emphasized at the beginning, this type of inversion procedure works only on slowly varying conductivity profiles. In other words, it can reproduce the overall shape of the conductivity profile, but not the rapidly changing local features. Therefore, appropriate smoothening was applied to the input AECC spectrum, which retained the characteristic overall shape of the AECC spectrum but greatly reduced its unevenness, as it is shown in Fig. 8(b). It should be mentioned, that such smoothening, e.g., a 3- or 5-point running average, is often used in experimental work to reduce the otherwise excessive random variations of the measured AECC spectrum. The iterative inversion technique was applied on the smoothened version of the input AECC profile using \( w = 2 \) as the scaling factor. This approach showed fairly good convergence up to the 3rd
iteration, after which the inversion became numerically unstable as it is shown in Fig. 8(a).

Fig. 9 shows the average and maximum differences between the inverted and exact conductivity profiles after increasing numbers of iteration steps. Based on these results, \( w = 2 \) and \( n = 1 \) or 2 are still the best choices for the scaling factor and the number of iterations even in the presence of random measurement noise. Although, a certain discrepancy will remain between the exact and inverted conductivity profiles at large depths unless the number of iterations is further increased, the agreement between them is relatively good up to the zero-crossing point at \( z = 0.4 \text{ mm} \), i.e., throughout the whole compressive part of the corresponding residual stress profile. It is worth mentioning here that the remaining discrepancy at larger depths is less significant from the point of view of life prediction; therefore it does not affect adversely the applicability of the proposed iterative inversion technique in NDE of shotpeened components.

4. Example of experimental application

In order to demonstrate the use of the proposed iterative procedure for inversion of experimental data, AECC measurements were conducted on an IN100 sample of Almen 8A peening intensity. Since the electric conductivity depth profile calculated from the input AECC spectrum cannot be directly verified by independent measurements, we had to rely on indirect verification between the residual stress profile estimated from the inverted conductivity profile using the independently measured piezoresistivity coefficient of the material and XRD results. Although, the ultimate goal of our present research effort is to predict the near-surface residual stress profile in the material, for the limited purposes of verifying the inversion procedure, a comparison between residual stress profiles is less than ideal because of the unknown and uncorrected contribution of cold-work to the actual electric conductivity variation. However, in the case of Almen 8A peening
intensity, the cold-work is fairly low and limited to a much shallower depth than the compressive residual stress.

Fig. 10(a) shows the relative AECC change measured on an IN100 specimen of Almen 8A peening intensity between 100 kHz and 30 MHz. First, the above described iterative procedure \((\nu = 2\) and \(n = 2\)) was used to reconstruct the depth-dependent electric conductivity profile. Then, the residual stress profile was estimated by assuming that the electric conductivity variation was entirely due to the piezoresistivity of the material. For isotropic plane-state of stress, the relative conductivity change can be calculated as follows \[16\]:

\[
\frac{\Delta \sigma}{\sigma_0} = \kappa_{ip} \frac{\tau_{ip}}{E},
\]

where \(\Delta \sigma\) is the stress-induced absolute change in the electric conductivity, \(\sigma_0\) is the intrinsic electric conductivity of the unstressed metal \((\sigma_0 = 1.33 \%\text{IACS})\), \(\tau_{ip}\) is the isotropic plane stress, \(E\) is Young’s modulus \((E = 210 \text{ GPa})\), and \(\kappa_{ip}\) is the dimensionless isotropic electroelastic coefficient, which is simply the sum of the parallel \(\kappa_{11}\) and normal \(\kappa_{12}\) electroelastic coefficients \((\kappa_{ip} = \kappa_{11} + \kappa_{12} = -1.58 \pm 0.16)\).

Fig. 10(b) shows the thereby reconstructed residual stress profile along with the XRD results that were independently measured on a different IN100 specimen of Almen 8A. Because of the low electric conductivity of nickel-base superalloys, capturing the peak compressive residual stress in moderately peened components will require further increasing the inspection frequency to 80–100 MHz, a challenging task that is currently underway in a parallel project. Although, the residual stress profile reconstructed from the eddy current data does not capture all the features exhibited by the XRD profile, this example illustrates that the proposed inversion procedure is suitable for adaptation in near-surface residual stress characterization. Part of the residual stress characterization problem is that, due to the square root relationship between inspection frequency and eddy current penetration depth, even a relatively wide frequency range of two and a half decades is insufficient to capture the peak conductivity as well as the baseline conductivity of the intact material in one single measurement. In addition, as we mentioned earlier, the accuracy of near-surface residual stress measurement is also adversely affected by the presently uncorrected cold-work contributions in the measured AECC spectrum, which are expected to be significant close to the surface, i.e., in the vicinity of the peak compressive stress.

5. Conclusions

Recently, it has been shown that in shot-peened nickel-base superalloys the depth profile of the protective compressive residual stress can be predicted by eddy current conductivity measurements. In order to estimate the near-surface residual stress profile \(\tau(z)\) from the measured AECC spectrum \(I(f)\), a two-step inversion procedure is needed. First, the electric conductivity profile \(\sigma(z)\) must be calculated from \(I(f)\) and then \(\tau(z)\) must be calculated from \(\sigma(z)\). This paper focused on the first step and proposed an improved iterative inversion procedure that eliminates some of the errors introduced by the previously used simplistic direct approach \[16\], which has become more and more apparent recently as both the inspection frequency range and the measurement accuracy increased.

According to the former simplistic inversion technique, it was crudely assumed that the eddy current distribution is localized at half of the standard penetration depth so that at any given frequency, the AECC corresponds directly to the actual electric conductivity at that depth. The proposed iterative inversion technique is based on the original simplistic inversion technique, but a feedback loop was introduced to increase its accuracy. This feedback loop exploits the fact that the forward electromagnetic problem, i.e., the calculation of the AECC spectrum from the electric conductivity depth profile, is fairly easy and
straightforward and a simple 1D approximation offers sufficient accuracy. Based on XRD evidence, a multiple-Gaussian near-surface conductivity profile was selected for verification purposes. The conductivity variation was limited to a shallow depth below the surface, which was much less than typical probe coil diameters.

The proposed iterative inversion technique was successfully validated by numerical simulations. It was shown that the proposed iterative inversion technique is numerically stable as long as the scaling factor remains in the range between \( w = 1.5 \) and 2, therefore it is reasonable to keep using \( w = 2.0 \) that was arbitrarily chosen in the simplistic inversion model [16]. It was also shown that four to five iterations are sufficient to achieve very good convergence toward the exact conductivity profile. However, the largest improvement with respect to the simplistic model is achieved in the first iteration and the second iteration is sufficient to achieve a reasonable inverted conductivity profile for all practical purposes.

The robustness of the proposed inversion procedure was further investigated by introducing random variations in the AECC spectrum to represent experimental uncertainties. It was shown that smoothing of the measured AECC profile is required to eliminate potential inversion instabilities. Moderate smoothing of the measured AECC spectrum does not affect the overall shape of the reconstructed conductivity profile and is a standard procedure to reduce random experimental variations. The proposed iterative inversion technique showed fast convergence and high accuracy that is approximately one order of magnitude better than that of the previously developed simpler method.

Considering the inherent experimental uncertainties associated with near-surface residual stress characterization in shot-peened nickel-base superalloys by eddy current conductivity measurements, the proposed inversion procedure is more than sufficient for practical purposes. However, further analytical and experimental studies are needed to better understand the coldwork contribution to the net conductivity variation in shot-peened components. It is worth mentioning here that LPB, and LSP are considered more sophisticated surface enhancement techniques since they produce roughly one order of magnitude lower cold-work levels, which makes them not only thermo-mechanically more stable, but also presents a better opportunity for eddy current residual stress assessment.

Acknowledgment

This work was performed at the University of Cincinnati in cooperation with the Center for NDE at Iowa State University with funding from the Air Force Research Laboratory on contract FA 8650-04-C-5228.

References