On the anomalously low attenuation of the leaky Rayleigh wave in a fluid-filled cylindrical cavity

Waled Hassan and Peter B. Nagy
Department of Aerospace Engineering and Engineering Mechanics, University of Cincinnati, Cincinnati, Ohio 45221-0070

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It is shown that the dispersive Rayleigh wave propagating around a concave cylindrical surface is substantially less attenuated by fluid loading than the corresponding wave on a flat surface. First, the analytical solution for vertically polarized shear wave scattering from a fluid-filled cylindrical cavity is formulated in the time domain, then the signal of interest is gated out and spectrum analyzed in order to numerically predict the attenuation caused by leakage into the fluid. On a concave surface the ratio of the normal and transverse displacement components produced by the circumferential creeping wave is lower than that of the ordinary Rayleigh wave propagating on a flat surface, which explains the reduced leaky attenuation caused by fluid loading. Experiments were carried out to verify these analytical predictions. The fluid-loading induced semicircumferential loss of the circumferential creeping wave around a cylindrical cavity was found to be in excellent agreement with the experimental measurements over a wide frequency range. © 1998 Acoustical Society of America. [S0001-4966(98)00609-2]

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INTRODUCTION

The work presented in this paper was initiated by recent efforts to use circumferential creeping waves for ultrasonic nondestructive testing of inaccessible airframe structures for fatigue cracking. One particular such application of great interest in the aerospace industry is the inspection of weep holes drilled through vertical stiffeners in wet-wing structures (used as fuel tanks) of both military and civilian aircraft in order to permit remnant fuel to be evenly distributed during flight. Unfortunately, they can become the sites from which fatigue cracks often originate. The most difficult cracks to be detected are the ones located at the upper part of the hole (12 o’clock position), away from the lower skin of the wing. The conventional ultrasonic creeping wave technique shown in Fig. 1(a) experiences a strong specular reflection from the near surface of the hole that masks the creeping wave signal that arrives later in time. Figure 1(b) shows the reflected signal from a 5-mm-diameter weep hole with and without a radial crack at 12 o’clock. This figure clearly demonstrates that the main problem with the conventional creeping wave inspection technique is the relatively low amplitude of the creeping wave signal with respect to the specular reflection which is, of course, not sensitive at all to the radial crack to be detected. In order to overcome this difficulty, a novel ultrasonic creeping wave technique was suggested. This new split-aperture technique shown in Fig. 2(a) resulted in the specular reflection and the creeping wave echoes being approximately equal in amplitude. Figure 2(b) illustrates the improved sensitivity of the split-aperture creeping wave technique in both pitch-catch and pulse-echo modes of operation. The outstanding sensitivity of the circumferential creeping wave technique in dry weep holes was well demonstrated on both simulated and real fatigue cracks. However, the presence of fuel or even its residues in the weep hole significantly affects the behavior of the circumferential creeping wave. It causes the creeping wave echo to split into a rather weak Rayleigh-type surface wave signal and a much stronger but somewhat slower halo signal. 2 In order to adapt the circumferential creeping wave inspection technique to the case of fluid-filled cavities it is important to understand the adverse effects of fluid loading on the Rayleigh-type circumferential surface wave, which is the only mode sensitive to such cracks.

The dispersion behavior of circumferential creeping waves around a fluid-filled cylindrical cavity in an infinite elastic medium was investigated in Ref. 3. Phase velocity, group velocity, as well as attenuation curves were presented. This frequency-domain analysis revealed some interesting aspects of circumferential wave propagation in a fluid-filled cylindrical cavity. However, this solution represents the overall behavior of the elastic field resulting from the interference of the two principal waves, namely the leaky Rayleigh and the halo waves, from which the propagation parameters of the leaky Rayleigh mode could not be isolated by itself. A time-domain analysis of the problem, in which the rf signals of the reflected field from a fluid-filled cylindrical cavity are calculated, would allow the separation of the leaky Rayleigh arrival from all other signals and would, therefore, permit an accurate study of the behavior of that specific mode.

Extensive literature is available on elastic waves propagating around solid cylinders and spheres immersed in fluid. 4–6 In these cases, the coupling between the fluid and the solid is relatively weak because of the usually large difference between their acoustic impedance. Of course much stronger acoustic coupling occurs when the solid rod or sphere is substituted by a thin shell. The problem of elastic waves running around air-filled elastic shells submerged in fluid has been extensively investigated by Gaunaurd, Werby, Überall, and others. 5–11 Kaduchak and Marston evaluated the...
surface displacement of a cylindrical shell insonified by plane waves perpendicular to the shell’s axis of symmetry. Optical visualization of the diffraction of a longitudinal pulse by a cylindrical cavity in an optically transparent solid was presented by Ying. Longitudinal wave scattering from fluid-filled cylindrical holes was studied experimentally in great detail by Sachse et al. However, in spite of the fairly strong acoustical coupling between the solid host and the fluid within the hole, their results did not show the presence of a Rayleigh-type surface mode propagating around the hole as the longitudinal mode in the solid is but very weakly coupled to this mode. We have found that vertically polarized shear waves are much more strongly coupled to the Rayleigh-type surface mode and generate relatively weaker whispering gallery modes in the fluid. Therefore a vertically polarized incident shear wave will be used in the present analysis.

A complete analytical solution to the problem of elastic wave scattering from cylindrical cavities was first obtained by White, whose work was later corrected by Lewis and Kraft. Gaunaurd and Überall studied elastic wave scattering from fluid-filled spherical cavities. The conventional approach to the solution of scattering problems involving simple geometries such as cylinders and spheres is to express the incident and the scattered waves in terms of a normal mode series. The expansion coefficients are determined by solving the associated boundary value problem. The complete solution is obtained, of course, by summing over all normal modes. However, because several modes may contribute simultaneously to the total scattering amplitude at a given frequency and because of the coupling between longitudinal and shear waves at the boundary, the results are complex and difficult to interpret. Haug et al. showed that partial wave expansion of the incident and scattered waves can be applied advantageously to the study of scattering of incident longitudinal waves from fluid-filled cavities. Solomon et al. extended the analysis of scattering from cylindrical cavities to include the case of incident shear waves. Our analysis takes advantage of this model to formulate the amplitude of the scattered shear wave in the frequency domain and then uses the inverse Fourier transform to calculate the rf signals in the time domain.

First, the time-domain solution of the problem of vertically polarized shear wave scattering from a fluid-filled cylindrical cavity in an elastic host is reviewed. The rf signal of the scattered shear wave is calculated for the case of a 6-mm-diam water-filled cavity in aluminum. The weak leaky Rayleigh wave is gated out from the waveform and its attenuation due to fluid loading is calculated by subtracting the spectra of two signals separated by a certain propagation distance. To establish confidence in this technique and to
numerically validate the results we first present attenuation calculations for two very well known cases, for which the exact solutions are readily available from the relevant dispersion equations. The two cases are (a) the case of a leaky Rayleigh wave propagating along the interface of a fluid half-space in contact with an elastic solid half-space, and (b) the case of a Rayleigh-type surface wave running around the inner surface of a free cylindrical cavity in an infinite elastic medium. Then, we calculate the attenuation of the leaky Rayleigh wave around the fluid-filled cavity and show that the leaky loss is substantially lower than expected based on the well-known leaky attenuation coefficient of the flat interface model. This anomalously low attenuation can be attributed to the fact that the aspect ratio between the normal and tangential displacement components of the elliptical surface trajectory is always smaller for the cylindrical geometry than for the frequency-independent flat case. Of course, in the high-frequency limit the aspect ratio in the curved geometry approaches that of the flat case. Finally we calculate the semicircumferential attenuation of the leaky Rayleigh wave around the fluid-filled cavity and show that the predicted result agrees very well with the experimentally measured semicircumferential attenuation.

I. THEORETICAL BACKGROUND

The analysis of vertically polarized shear wave scattering from a fluid-filled cylindrical cavity was carried out by Solomon et al. The results presented here are adapted from their paper. As shown in Fig. 3, we consider the problem of a vertically polarized shear (SV) wave normally incident on an infinitely long cylindrical cavity of radius $a$, which is aligned with the $z$ axis. The cavity may be free or contain an inviscid fluid with density $\rho_f$ and sound speed $c_f$. The elastic host is characterized by its density $\rho_s$ and its shear and dilatational wave velocities $c_s$ and $c_d$, respectively. The SV plane wave incident along the $x$ axis can be expressed as a partial-wave series,

$$\phi_s = \phi_0 e^{ik_s x} = \phi_0 \sum_{n=0}^{\infty} e_n i^n J_n(k_s r) \cos(n \theta),$$

where $k_s = \omega/c_s$ is the shear wave number, $J_n$ denotes the $n$th-order Bessel function of the first kind, and the Neumann factor is $e_0 = 1$ and $e_n = 2$ for $n \neq 1$. For brevity, the common time factor $e^{-i\omega t}$ has been omitted from Eq. (1) and from all subsequent equations. The scattered dilatational and shear waves in the solid host must be of the form

$$\phi_s = \sum_{n=0}^{\infty} e_n i^n A_n H_n^{(1)}(k_s r) \sin(n \theta),$$

and

$$\psi_s = \sum_{n=0}^{\infty} e_n i^n B_n H_n^{(1)}(k_s r) \cos(n \theta),$$

respectively, where $k_s = \omega/c_s$ is the dilatational wave number and $H_n^{(1)}$ denotes the $n$th-order Hankel function of the first kind. In addition, in a fluid-filled cavity a standing compressional wave may be excited in the form given by

$$\phi_f = \sum_{n=0}^{\infty} e_n i^n C_n J_n(k_f r) \sin(n \theta),$$

where $k_f = \omega/c_f$ is the wave number in the fluid. In Eqs. (1)–(4) $\phi$ and $\psi$ represent the scalar and vector displacement potentials, respectively. The expansion coefficients $A_n$, $B_n$, and $C_n$ can be determined from the appropriate boundary conditions. For the fluid-filled hole these are the continuity of the normal displacement and stress and the vanishing of the shear stress at the cavity wall. In comparison, for the free hole both normal and tangential stresses must vanish at the surface.

Using the inverse Fourier transform we can write the scattered vertically polarized shear potential in the time-domain as follows:

$$\psi_s(r, \theta, t) = \int_{-\infty}^{\infty} \left( \sum_{n=0}^{\infty} e_n i^n B_n H_n^{(1)}(k_s r) \cos(n \theta) \right) e^{-i\omega t} \, d\omega,$$

where we took the negative sign convention in the exponential for the inverse Fourier transform. A far-field approximation is obtained by substituting the high-frequency asymptote of the Hankel function of the first kind as follows

$$\psi_s(r, \theta, t) \approx \sqrt{\frac{2}{\pi}} \int_{-\infty}^{\infty} \sqrt{\frac{\omega}{\omega_0 e^{i(\pi/4 - \omega t)}}} B_0 + 2 \sum_{n=1}^{\infty} i^n B_n e^{-n\pi^2\omega^2} \cos(n \theta) \bigg) \, d\omega,$$

Equation (6) can be used to numerically calculate the rf waveform representing the scattered shear potential from the cylindrical cavity for both the fluid-filled and the dry cases.

II. NUMERICAL RESULTS

In order to demonstrate how the numerically determined scattering waveforms can be further processed to obtain the attenuation coefficient of individual wave modes, we first present attenuation calculations for two very well known cases for which the exact solutions are readily available from the relevant dispersion equations. In the first example, we
calculate the leaky attenuation of the Rayleigh wave propagating on a fluid-loaded solid half-space. In this case the surface wave is nondispersive and the leakage occurs into the fluid. In the second example, we calculate the leaky attenuation of the Rayleigh wave running around the inner surface of a free cylindrical cavity in an infinite elastic medium. In this case the surface wave is dispersive and the leakage occurs into the solid. Finally, we use this technique to solve the problem of main interest in this paper, i.e., to calculate the fluid-loading induced attenuation of the Rayleigh wave running around the inner surface of a fluid-filled cylindrical cavity in an infinite elastic medium. In this case the surface wave is dispersive and leaks into both the fluid and the solid.

The exact attenuation of this mode cannot be directly obtained from the dispersion equation which gives the apparent attenuation of the complex interference field dominated by two principal components, namely the leaky Rayleigh and the halo modes. To the best of our knowledge the exact attenuation of the leaky Rayleigh mode around a fluid-filled cylindrical cavity has never been calculated before and is not available in the literature. In all the numerical calculations presented below, the radius of the cylindrical cavity is taken as \(r = 5\) mm, the fluid is taken as water with density \(\rho_f = 1000\) kg/m\(^3\) and sound velocity \(c_f = 1494\) m/s. The elastic solid is taken as aluminum with density \(\rho_s = 2700\) kg/m\(^3\), shear velocity \(c_s = 3140\) m/s, and longitudinal velocity \(c_d = 6380\) m/s.

### A. Fluid-loaded solid half-space

The Green’s function for a system consisting of a two-dimensional, impulsive, monopole line source in a fluid/solid configuration with a plane geometry was formulated by de Hoop et al. using the modified Cagniard technique. The pulse corresponding to the leaky Rayleigh mode can be readily gated out from the Green’s function and spectrum analyzed. By varying the distance between the transmitter and the receiver we can compute the attenuation of the leaky Rayleigh wave propagating along the interface.

Figure 4 shows the space-time Green’s function for the reflected acoustic pressure from a water/aluminum interface as a function of time for five transmitter/receiver distances, \(d\). The transmitter and the receiver were placed infinitesimally close to the water/aluminum interface. The later arriving but very strong compressional wave was slightly clipped to better display the leaky Rayleigh pulse. All the waves present in the rf waveform for this case along with their times of arrival were positively identified and thoroughly described by de Hoop et al. A flat-top window was used to isolate the leaky Rayleigh arrival from all the other modes. The attenuation coefficient was then calculated as

\[
\alpha_{\text{plane}} = \frac{20(\log[G_1(\omega)] - \log[G_2(\omega)])}{d_2 - d_1},
\]

where \(G_i(\omega)\) is the magnitude of the spectrum of the leaky Rayleigh signal at a distance \(d_i\) from the transmitter. Figure 5 shows the attenuation coefficient for the leaky Rayleigh wave along the plane water/aluminum interface as a function of frequency. The exact solution was calculated from the exact dispersion equation. We notice that the time-domain solution is identical to the exact solution except for the numerical difficulties associated with calculating the spectrum of the time-domain signal using FFT, which cause the technique to fail at high frequencies as it is evident from the figure.

### B. Fluid-free cylindrical cavity

In order to facilitate the adaptation of our numerical technique to dispersive waves propagating on curved surfaces using Eq. (6), a bandlimited excitation was imposed to restrict the spectral components over which the integration is performed. This allowed the replacement of the integration by a finite sum. Figure 6(a) shows the bandlimited excitation used in the calculations. It is given analytically as

\[
f(t) = \left[1 - \cos\left(\frac{2\pi t}{T}\right)\right]\sin\left(\frac{2\pi t}{T}\right),
\]

FIG. 4. Space-time Green’s function for the reflected acoustic pressure in water from a water/aluminum interface as a function of time for five transmitter/receiver distances.

FIG. 5. Attenuation coefficient of the leaky Rayleigh wave propagating along a flat water/aluminum interface.
where \( T = 0.09 \, \mu s \) represents the total duration of the excitation pulse. Figure 6(b) shows the band-limited frequency spectrum of the excitation signal. The center frequency and \(-3\, \text{dB} \) bandwidth are both approximately 11 MHz. It should be mentioned that the spectra of the incident and scattered signals were calculated with a relatively high resolution of 500 Hz in order to avoid folding back of the calculated waveforms up to 1 ms.

Figure 7 shows the rf signals representing the backscattered vertically polarized shear potential from a fluid-free cylindrical cavity in an infinite aluminum host for angles between \( 40^\circ \) and \( 180^\circ \) degrees calculated at every \( 10^\circ \) with the receiver at a distance of \( r = 3a \) from the center of the cavity. It should be mentioned that Eq. (6) represents a far-field approximation of the scattered field therefore the distance of the receiver from the cavity does not affect the calculated waveform except for a \( t_r = 3al/c_s \) constant time delay. These results were calculated using the same parameters mentioned previously. Again, to better display the Rayleigh arrival the stronger specular reflection was slightly clipped. The first of the three arrivals in the rf signal represents the specularly scattered shear wave, the second is the counterclockwise propagating circumferential Rayleigh wave with the shorter path and the third is the clockwise propagating circumferential Rayleigh wave with the longer path. The time of arrival of each one of these signals can be calculated from simple ray tracing considerations. Figure 8 shows a schematic diagram of the specular reflection and the earlier of the two Rayleigh arrivals. The two Rayleigh signals are generated at diametrically opposite points where the incident shear wave grazes the circumference of the cavity. The shorter-path Rayleigh arrival travels around the cavity counter-clockwise through an angle \( \theta \) equal to the azimuthal angle of the receiver, whereas the longer-path Rayleigh pulse travels clockwise through an angle of \( 2\pi - \theta \). From Fig. 8, we can write the time of arrival for each one of these signals as follows:

\[
t_{\text{spec}} = t_r - \frac{2a}{c_s} \sin \frac{\theta}{2} ,
\]

\[
t_{\text{Rsp}} = t_r + \frac{a \theta}{c_R} ,
\]

\[
t_{\text{Rlp}} = t_r + \frac{a(2\pi - \theta)}{c_R} ,
\]

where \( t_{\text{spec}} \) is the time of arrival of the specularly scattered wave, and \( t_{\text{Rsp}} \) and \( t_{\text{Rlp}} \) are the times of arrival of the short-path and long-path Rayleigh signals, respectively. For the sake of simplicity, \( c_R \) is taken as the velocity of the Rayleigh wave on the free surface of the solid half-space. The predic-

FIG. 6. (a) The single-cycle waveform of the excitation pulse used in calculating the rf signals in the cylindrical geometry and (b) its bandlimited frequency spectrum.

FIG. 7. The back scattered vertically polarized shear potential from a 6-mm-diameter fluid-free cylindrical cavity in an infinite aluminum host for angles between \( 40^\circ \) and \( 180^\circ \) calculated at every \( 10^\circ \).

FIG. 8. Schematic diagram illustrating the first two scattered waves from a fluid-free cylindrical cavity in an infinite elastic host. Ray tracing is used to predict the times of arrival of the different signals.
tions from Eqs. (9a)–(9c) are in very close agreement with the results obtained numerically.

The leaky attenuation caused by the curvature of the surface is measured in terms of the normalized semicircular loss $L_p$, which is defined as the total attenuation halfway around the cavity

$$L_p = 20\log\left(\frac{G_1(\omega)}{G_2(\omega)}\right)_{\theta_2 - \theta_1},$$

where $G_i(\omega)$ is the magnitude of the spectrum of the windowed Rayleigh signal at angle $\theta_i$. Figure 9 shows the semicircular loss of the Rayleigh wave running around the inner surface of a fluid-free cylindrical cavity in an infinite solid as a function of the normalized frequency $ak_s$. The solid line represents the exact solution as calculated from the dispersion equation. The circles represent the semicircular loss calculated from the time-domain solution. The Rayleigh arrival was gated out from the time-domain signal obtained at the two locations of $\theta_1 = 30^\circ$ and $\theta_2 = 60^\circ$. A flat-top window was applied to the gated signals and the spectra of the resulting signals were calculated. Figure 9 shows that the time-domain solution is in excellent agreement with the exact solution from the dispersion equation over the normalized frequency range of 15–120. However, the time-domain solution seems to be breaking down for normalized frequencies below 15 and above 120 due to the inherent numerical difficulties involved in the calculations of the spectra at very low and very high frequencies.

C. Fluid-filled cylindrical cavity

For the case of a fluid-filled cavity in an elastic medium, the results are obtained in a manner similar to that used in the case of the fluid-free cavity. However, the results are much more complex due to the presence of the fluid and have to be analyzed more carefully. Figure 10 shows the backscattered vertically polarized shear potential from a 6-mm-diam water-filled cylindrical cavity in an infinite aluminum host for angles between $\theta = 40^\circ$ and $\theta = 180^\circ$ calculated at every $10^\circ$. Again, the receiver is placed at a distance of $r = 3a$ from the center of the cavity. The first arrival is the specularly scattered shear wave, followed by the weak circumferential Rayleigh-type creeping wave and a much stronger arrival through the fluid filling the cavity. The latter one is actually a combination of two modes that arrive at almost the same time. One is the direct transmission through the fluid which becomes weaker and weaker as the angle of scattering increases. For angles larger than $\theta = 30^\circ$ there is a second mode, the so-called halo wave, caused by the leakage of the Rayleigh mode into the fluid. According to the classical physical explanation based on ray theory, halo modes are produced by the repeated reflection of the compressional bulk wave in the fluid at the cylindrical wall. For a water/aluminum combination the signal going through the fluid is dominated by the direct transmission up to about $\theta = 60^\circ$ while the halo mode becomes the stronger one at higher angles. It is worth mentioning that another rather weak arrival that propagates through part of its path as a dilatational skimming wave around the cavity can also appear between the leaky Rayleigh and halo waves depending on the combination of the sound velocities in the fluid and the solid. The arrival times for the specular and the leaky

FIG. 11. Schematic diagram illustrating the path of the direct bulk transmission in a fluid-filled cylindrical cavity in an elastic medium.
Rayleigh modes can still be calculated from Eqs. (9a)–(9c). As for the direct bulk transmission mode, the simple ray tracing schematic diagram shown in Fig. 11 leads to the following expression for its time of arrival,

\[ t_f = t_r - a \left[ \frac{\cos(\delta) + \cos(\theta - \delta + 2\theta_f)}{c_s} - \frac{2\cos(\theta_f)}{c_f} \right], \tag{11} \]

where \( \delta = \sin^{-1}(\zeta/d) \) and \( \theta_f = \sin^{-1}(c_f/c_s \sin \delta). \) The appropriate value of \( \zeta \) for any given scattering angle \( \theta \) is obtained by requiring that the emerging ray subtends angle \( \theta \) to the incident direction. Figure 12 illustrates the path taken by the halo mode. Its arrival time can be calculated as follows:

\[ t_h = t_r + a \left[ \frac{\theta - 2\theta_R}{c_R} + \frac{2\cos(\theta_R)}{c_f} \right], \tag{12} \]

where \( \theta_R = \sin^{-1}(c_f/c_R) \) is the Rayleigh angle, which is approximately 30° for the water/aluminum combination.

Figure 13 shows the fluid-loading induced loss of the leaky Rayleigh circumferential creeping mode around a fluid-filled cylindrical cavity for different propagation angles as a function of the normalized frequency \( ak_s \). At a certain angle \( \theta \) the rf signals representing the backscattered field are computed for the cases of the water-free and water-filled cylindrical cavities in aluminum. The Rayleigh-type creeping wave is then gated out from the time-domain signals and a flat-top window is applied. Finally, the spectrum for each case is calculated and the fluid-loading induced loss is obtained as

\[ L_{\theta}(\omega) = 20 \log \left( \frac{G_{\theta \omega}^{\text{fluid}}(\omega)}{G_{\theta \omega}^{\text{free}}(\omega)} \right), \tag{13} \]

where \( G_{\theta \omega}^{\text{fluid}}(\omega) \) and \( G_{\theta \omega}^{\text{free}}(\omega) \) are, respectively, the magnitudes of the spectra of the Rayleigh wave signal with and without fluid in the cavity. It is worth noting that the change in the fluid-loading induced loss is the largest when the scattering angle is changed from 30° to 60°. As the propagation angle is increased through 180°, the difference becomes more uniform. This indicates that the fluid-loading loss does not change strictly linearly with the scattering angle, i.e., the propagation angle of the surface wave around the cylindrical cavity. Nevertheless, a linear approximation for the dependence of the fluid-loading loss on the propagation angle does not introduce significant error in the results and may be taken as an approximation.

Of course the dispersive leaky Rayleigh wave propagating around a fluid-free cylindrical cavity asymptotically approaches, in the high frequency limit, the true Rayleigh wave along the flat interface of an elastic half-space. Similarly, it is expected that the ‘‘doubly-leaky’’ Rayleigh wave propagating around a fluid-filled cylindrical cavity also asymptotically approaches the leaky Rayleigh wave along a fluid-loaded elastic half-space. This asymptotic behavior is not obvious from Fig. 13 which shows the total fluid-loading induced loss of the Rayleigh-type surface wave over given angles of propagation rather than the leaky attenuation coefficient itself. Figure 14 shows the leaky attenuation coefficient due to fluid-loading in the two cases of flat and cylindrical geometry as a function of the normalized frequency. The dashed line represents the plane approximation whereas the solid line represents the leaky coefficient in the cylindrical geometry.
used in nondestructive evaluation applications. It was possible to measure the fluid loading induced losses at two different scattering angles according to Eq. (14). The second effect, which dominates the total fluid-loading induced loss at all but very low angles, is that the leaky attenuation is actually lower on a curved surface than on a flat one (\( \alpha_{\text{curved}} \)) than on a flat one (\( \alpha_{\text{plane}} \)).

Since the leaky attenuation is solely due to the normal vibration component of the surface in contact with the fluid, it seems to be reasonable to assume that the aspect ratio of the elliptically polarized particle displacement must be lower on the curved surface than on the flat one. It is well known that a surface particle under the influence of a Rayleigh wave undergoes a counterclockwise elliptically polarized motion. For the classical Rayleigh wave propagating on the free surface of a solid half-space, this aspect ratio is a function of Poisson’s ratio only. For example, in the case of aluminum with Poisson’s ratio of \( \nu = 0.34 \), this ratio is approximately 1.6. For guided waves in general, e.g., for rod and plate modes, the aspect ratio is also dependent on frequency. The role of this aspect ratio in determining the degree of leakage from the surface has been recently investigated by Nagy and Kent25 and Nagy and Nayfeh.26,27 As this aspect ratio increases, i.e., the normal component of the displacement becomes larger compared to the tangential component, the leaky loss increases. In contrast, the leaky loss decreases as the tangential displacement component increases with respect to the normal component.

Figure 16 shows the aspect ratio of the elliptical trajectory of the particle’s motion for the case of a Rayleigh wave along the free surface of an infinite elastic half-space and the case of the dispersive Rayleigh-type wave around the free surface of a cylindrical cavity in an infinite elastic solid. For the cylindrical geometry the exact aspect ratio can be readily obtained from Rulf’s results.24 It is clear from Fig. 16 that the aspect ratio in the cylindrical geometry is a function of frequency whereas it is frequency independent in the flat geometry. It is also clear that this aspect ratio is smaller in the cylindrical geometry than it is in the flat case, and that it approaches its high-frequency flat asymptotic limit from below. This means that the normal displacement component in
circumferential loss of the leaky Rayleigh mode was calculated theoretically and verified experimentally. It was observed that the calculated and measured semicircumferential loss was significantly less than predictions obtained from the flat interface model. The difference between the two cases can be mainly attributed to the reduced ratio of the normal and tangential surface displacement components, which represents the aspect ratio of the particle’s elliptical trajectory. It was found that for a curved surface this ratio is less than the one associated with the flat case. Since the leaky attenuation is caused solely by the normal displacement component we can conclude that the leaky attenuation of the Rayleigh-type surface wave propagating on a curved surface is inherently lower than the one on a plane surface.

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