A simple numerical model of the apparent loss of eddy current conductivity due to surface roughness

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Abstract

In light of its frequency-dependent penetration depth, the measurement of eddy current conductivity has been suggested as a possible means to allow the nondestructive evaluation of subsurface residual stresses in shot-peened specimens. This technique is based on the so-called electro-elastic effect, i.e. the stress-dependence of the electrical conductivity. Unfortunately, the relatively small change in electrical conductivity caused by shot peening is often distorted, or even completely overshadowed, by the apparent conductivity loss caused by the accompanying surface roughness. This geometrical artifact is due to the fact that as the frequency increases, and therefore the penetration depth decreases, the path of the eddy current must follow a more tortuous route in the material, which produces a reduction in the observed conductivity. This paper addresses the apparent reduction of the near-surface electrical conductivity measured by the eddy current method in the presence of surface roughness. The rough surface is modeled as a one-dimensional sinusoidal corrugation using the Rayleigh–Fourier method. The apparent conductivity is determined from the resulting change in the plane-wave reflection coefficient of the conducting half-space at normal incidence. In spite of the simplicity of the suggested analytical model, the obtained theoretical results are found to be in good qualitative agreement with recently published experimental data from shot-peened copper specimens.

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1. Introduction

It is a well-known fact that surface properties are very important in determining the performance of structural components. In most cases of long-term failure of fracture-critical components, the underlying factor is the presence of tensile stresses, which contribute to both crack initiation and rapid crack growth, ultimately causing material and hence component failure. Near-surface compressive residual stresses mitigate the adverse effects of in-service tensile stresses and thereby can significantly extend the fatigue life of structural components. One of the best and most inexpensive ways of inducing compressive stresses in materials is by shot peening, which is a cold working process. Shot peening not only produces a thin layer of beneficial compressive residual stresses just beneath the material surface, but also leaves a plastically deformed near-surface region as well as an undesirable surface roughness. The penetration depth, and to a lesser degree, the absolute level of the induced compressive residual stresses can be increased by increasing the peening intensity. Unfortunately, increasing peening intensity also increases the level of remnant cold work in the material, which dramatically reduces the stress relaxation temperature thereby causing accelerated decay of the protective residual stresses at elevated temperatures. In addition, increasing peening intensity also increases the accompanying surface roughness, which produces adverse stress concentrations and could even lead to the reduction of service life in the presence of significant thermo-mechanical relaxation.

The need for reliable quality control during surface treatment and, especially, the inherent instability of the residual stress during long-term service at elevated temperatures require the development of nondestructive materials characterization methods capable of quantitatively assessing the remaining level and penetration depth of the residual stresses in shot-peened specimens. In light of its frequency-dependent penetration depth, the measurement of eddy current conductivity has been suggested as a possible
means to allow the nondestructive evaluation of subsurface residual stresses in shot-peened specimens. This technique is based on the so-called electro-elastic effect, i.e. the stress-dependence of the electrical conductivity. Unfortunately, the relatively small change in electrical conductivity caused by shot peening is often distorted, or even completely overshadowed, by the apparent conductivity loss due to the accompanying surface roughness. This geometrical artifact is caused by the fact that as the frequency increases, and the eddy current penetration depth decreases, the path of the eddy current must follow a more tortuous route in the material, which produces a perceivable reduction in the observed conductivity.

Recently, it has been suggested that eddy current electrical conductivity measurements might provide a viable nondestructive evaluation method for residual stress assessment in shot-peened components [1–3]. However, this claim was not sufficiently substantiated by experimental evidence and subsequent studies came to somewhat contradictory conclusions. For example, Lavrentyev et al. found that the electrical conductivity of 7075 Aluminum was not sufficiently sensitive to stress to allow nondestructive residual stress assessment in shot-peened specimens, but might be useful for cold work measurements [4]. The authors also found that the measured conductivity loss at higher frequencies and increasing peening intensities could be, to a large degree, caused by the induced surface roughness, although they did not investigate this effect in depth. It is clear from these results that various factors, such as hardness, texture and surface roughness, will have to be isolated while performing nondestructive evaluation of the residual stress in shot-peened components. Amongst these, the factor that might create the largest error in thermomechanically relaxed components is surface roughness. It has been found that residual stress relaxation in shot-peened specimens is accompanied by a more or less proportional decay of the cold work effects (mainly increased dislocation density and anisotropic texture), but the surface roughness remains essentially unaffected and might even increase as a result of corrosion, erosion, or fretting wear.

Recent experimental results indicate that when such thermally relaxed specimens are subjected to eddy current inspection, there is a considerable reduction in the measured electrical conductivity, which is due to the continued presence and increasing dominance of surface roughness [5]. It has been found that the apparent electrical conductivity decreases with increasing peening intensity and inspection frequency even in fully recrystallized specimens, i.e. when both residual stress and cold work effects are entirely eliminated, therefore the true electrical conductivity of the specimen recovers its original intact value. This apparent reduction in electrical conductivity is due to the fact that the topography of the shot-peened surface remains irregular and exhibits a relatively high surface roughness. Since at high inspection frequencies the eddy current closely follows the contour of the surface, it has to travel longer distances than in the case of smooth surfaces. Due to this increase in the path length of the eddy current loop, the apparent resistance of the material increases, i.e. the apparent electrical conductivity decreases. We use the term 'apparent' because the actual conductivity of the recrystallized material is the same as that of the original intact specimen and the lower value of conductivity measured by eddy current inspection is a geometrical artifact due to the presence of surface roughness.

These conclusions have been experimentally verified by Blodgett et al. in shot-peened copper specimens [5]. Due to the high conductivity of copper, the penetration depth at a given inspection frequency is relatively low. At the same time, due to the high ductility of copper, the surface roughness produced by a given peening intensity is relatively large. As a result, the surface roughness induced apparent loss of electrical conductivity in copper could easily reach 20–25% at maximum inspection frequencies of 10 MHz and is readily measurable. In high-strength, high-temperature engine materials, like titanium alloys and nickel-base superalloys, the electrical conductivity is only around 1–2% IACS and consequently the eddy current penetration depth is fairly high, approximately 150–200 μm at 10 MHz. Even after accounting for the fact that typical peening intensities tend to result in approximately 2–3 times higher roughness in ductile copper (5–10 μm RMS) than in much harder engine materials (2–4 μm RMS), one has to conclude that the expected underestimation of the electrical conductivity in the presence of surface roughness will be comparable to the expected less than 1% conductivity change due to compressive near-surface residual stresses, which indicates that the described artifact will significantly affect eddy current inspection of shot-peened turbine engine alloys. It should be further emphasized that the ratio between the actual conductivity change due to the presence of residual stresses and the spurious apparent conductivity loss caused by surface roughness significantly decreases during thermomechanical relaxation. Therefore, the described artifact plays a more important role in the eddy current evaluation of engine components after long-term service than directly after surface treatment.

Generally, a two-dimensional randomly rough surface can be regarded as a superposition of sinusoidal corrugations of random phase and amplitude distributions to properly account for the statistical features of the surface topography. As a first step towards developing an accurate predictive model for random surfaces, in this paper we present a highly simplified analytical approach that allows the estimation of the surface roughness induced apparent conductivity loss based on an equivalent one-dimensional sinusoidal corrugation. First, we present an approximate solution for the roughness induced conductivity loss of surfaces with a simple sinusoidal corrugation based on the well-known Rayleigh–Fourier method. Second, we derive
explicit high-frequency asymptotic expressions for the reduced conductivity using the Kirchhoff approximation and demonstrate that for slight corrugations the numerical solution based on the Rayleigh–Fourier method converges to the Kirchhoff approximation. Third, we develop a very simple explicit ad hoc expression that well approximates the numerical predictions of the Rayleigh–Fourier method over the whole frequency range and therefore can be used as a practical tool to directly estimate the apparent conductivity loss for a sinusoidal corrugation of known amplitude and periodicity. Finally, we compare the predictions of this simple approximation to previously published experimental data and show that the model qualitatively predicts both the absolute value and the frequency dependence of the apparent conductivity loss for a sinusoidal corrugation of known amplitude and demonstrate that for slight corrugations the numerical predictions of the Rayleigh–Fourier method converge and show that the model qualitatively predicts both the absolute value and the frequency dependence of the apparent conductivity loss and point out the need for future research efforts to further increase the quantitative accuracy of the method.

2. Theory

In this section we present the theory of electromagnetic wave reflection from a sinusoidally corrugated rough surface at normal incidence. The solution is based on the Rayleigh hypothesis that stipulates that the scattered field is a sum of outgoing plane waves and the unknown amplitudes of the different diffraction orders are determined by applying the relevant boundary conditions on the surface [6–8]. The main restrictions of this method are that only outgoing scattered waves are considered and that the expansion in series converges only if the surfaces are ‘slightly rough,’ i.e. the local slope of the surface remains sufficiently small everywhere. Interestingly, it has been showed numerically on specific problems that the Rayleigh method yields accurate results well beyond the expected domain of validity of the Rayleigh hypothesis [9]. It has also been proved that for analytical surface profiles the applicability of the numerical Rayleigh method is much more extensive than the validity of the Rayleigh hypothesis [10].

Let us consider the phenomenon of plane wave reflection and transmission at a corrugated surface of sinusoidal profile between a dielectric and a conductive half-space at normal incidence. Fig. 1 shows a schematic diagram of the scattering of a normally incident plane wave at a corrugated surface (Φ could be either the electric or magnetic field or, e.g. the vector potential). The surface profile is assumed to be harmonic

\[ s(x) = a \cos(Kx), \]

where \( a \) denotes the amplitude of the surface corrugation, \( K = 2\pi/\Lambda \), and \( \Lambda \) is the period of the corrugation. A normally incident plane wave \( \Phi' \) of transverse magnetic (TM) polarization produces mutually orthogonal magnetic \( \mathbf{H}' \) and electric \( \mathbf{E}' \) fields given by

\[ \mathbf{H}' = a_y e^{i(kz-i\omega t)} \text{ and } \mathbf{E}' = a_z \eta_0 e^{i(kz-i\omega t)}. \]

Here, \( t \) denotes time, \( \omega \) is the angular frequency, \( \eta_0 = \sqrt{\mu_0/\varepsilon_0} \) is the intrinsic impedance of a vacuum, and \( k = \omega/\sqrt{\mu_0\varepsilon_0} \) is the electromagnetic wavenumber in a vacuum, where \( \varepsilon = 1/\sqrt{\mu_0\varepsilon_0} \) is the wave speed and \( \lambda \) is the wavelength. Generally, the reflected field \( \Phi' \) can be expressed as the sum of an infinite series

\[ \Phi' = \sum_{n=-\infty}^{\infty} \Phi'_n, \]

where all but the zeroth-order diffracted wave \( \Phi'_0 \) are evanescent since \( \lambda \gg \Lambda \). The associated magnetic and electric fields are given by

\[ \mathbf{H}'_n = R_n a_y e^{i(\kappa nKx - \kappa n\omega t)} \text{ and } \mathbf{E}'_n = \eta_0 R_n a_z e^{i(\kappa nKx - \kappa n\omega t)}, \]

where the polarization direction of the reflected electrical field is

\[ a'_n = -\frac{\kappa_n}{\kappa} a_y - \frac{nK}{\kappa} a_z, \]

and the wave number components are related through

\[ \kappa^2 = n^2K^2 + \kappa_n^2. \]

Similarly, the transmitted field \( \Phi' \) can be also expressed as the sum of an infinite series

\[ \Phi' = \sum_{n=-\infty}^{\infty} \Phi'_n \]
\[ \Phi' = \sum_{n=-\infty}^{\infty} \Phi'_n. \]

The associated magnetic \( \mathbf{H}'_n \) and electric \( \mathbf{E}'_n \) fields are given by

\[
\mathbf{H}'_n = T_n \mathbf{a}_x \ e^{i(nKx + \kappa_n z - \omega t)}
\]
and
\[
\mathbf{E}'_n = T_n \mathbf{a}_x \ e^{i(nKx + \kappa_n z - \omega t)},
\]
where \( \zeta = \sqrt{-\mu \sigma / \epsilon} \) is the intrinsic impedance of the conductor,

\[
a'_n = \frac{k_n}{k} a_x - \frac{nK}{k} a_x,
\]
and

\[ k^2 = n^2 K^2 + \kappa_n^2 = \mu \sigma. \]

It should be mentioned that all the transmitted modes are evanescent since the wavenumber \( k \) in the conductor is a complex number with equal real and imaginary components.

These formal solutions for the incident \( \Phi' \), reflected \( \Phi'' \), and transmitted \( \Phi' \) fields automatically satisfy the wave equation in their respective media. At the corrugated boundary \( z = s(x) \), they also have to satisfy the relevant boundary conditions, namely the continuity of the tangential components of the electric and magnetic field intensities and the continuity of the normal components of the electric and magnetic flux densities. However, these boundary conditions are redundant and the sought reflection and transmission coefficients can be calculated simply by satisfying the continuity of the tangential field components. It should be mentioned that the complete symmetry of the problem for \( s, n \rightarrow -s, -n \) transformation assures that \( T_n = T_{-n} \) and \( R_n = R_{-n} \), which will be exploited later to simplify the numerical solutions.

2.1. Boundary conditions

The continuity of the tangential component of the magnetic field intensity can be expressed as follows

\[
\mathbf{H}'[x, s(x)] + \mathbf{H}''[x, s(x)] = \mathbf{H}''[x, s(x)]
\]
or simply (without the common \( e^{-i\omega t} \) term)

\[
e^{i\omega t} + \sum_{n=-\infty}^{\infty} R_n e^{i(nKx + \kappa_n s(x))} = \sum_{n=-\infty}^{\infty} T_n e^{i(nKx + \kappa_n s(x))}.
\]

The continuity of the tangential component of the electric field intensity can be expressed as follows

\[
t(x) \cdot \mathbf{E}'[x, s(x)] + t(x) \cdot \mathbf{E}''[x, s(x)] = t(x) \cdot \mathbf{E}''[x, s(x)],
\]
where \( t(x) \) denotes the unit vector tangential to \( s(x) \)

\[
t(x) = \frac{1}{\sqrt{1 + \left( \frac{ds}{dx} \right)^2}} \mathbf{a}_x + \frac{ds}{dx} \mathbf{a}_x,
\]
so that we get

\[
\eta_0 e^{i\omega c s(x)} = \eta_0 \sum_{n=-\infty}^{\infty} R_n \left( \frac{k}{k_n} + \frac{nK}{k} \frac{ds}{dx} \right) e^{i(nKx - \kappa_n s(x))}
\]
and

\[
\zeta \sum_{n=-\infty}^{\infty} T_n \left( \frac{k}{k_n} - \frac{nK}{k} \frac{ds}{dx} \right) e^{i(nKx - \kappa_n s(x))}.
\]

As we mentioned earlier, the main restriction of the Rayleigh hypothesis is that only outgoing reflected and transmitted plane waves are considered (see Eqs. (4) and (8)), which is strictly valid only beyond the thin interface region containing the rough surface. Whenever the maximum slope of the surface reaches significant values, forward propagating reflected waves will also be present within this interface region. Of course these waves will ultimately modeconvert into outgoing reflected and transmitted waves via multiple scattering, but the fact that they are neglected in the boundary conditions inherently limits the applicability of the Rayleigh method to surfaces of small slope, i.e. small \( a/l \) ratios in the case of harmonic corrugations. A rough estimate on the allowable maximum amplitude-to-period ratio can be obtained based on ray approximations. Forward reflection occurs when the maximum slope at the inflection point reaches 45°, i.e. at \( a/K = 1 \) or \( a/l = 0.16 \). Actually, numerical studies have shown that accurate results can be obtained by this technique up to as high as \( a/l = 0.2-0.3 \) [9,10]. In the following calculations, the \( a/l \) ratio will be limited to a much lower value of 0.05, therefore the analytical results satisfying the boundary conditions of Eqs. (12) and (15) will be referred to as 'exact' (within the limitations of the Rayleigh hypothesis). Of course, for computational reasons, the infinite series in these equations will have to be truncated at a finite number of diffraction orders, therefore the calculated results will be referred to as 'numerical' solutions.

2.2. Numerical solution for sinusoidal surfaces

After substituting the surface profile from Eqs. (1), (12) and (15) we get the following two boundary condition equations for a sinusoidal surface

\[
e^{i\omega a \cos(Kx)} + \sum_{n=-\infty}^{\infty} R_n e^{-i\kappa_n a \cos(Kx)} e^{inKx}
\]
and

\[
= \sum_{n=-\infty}^{\infty} T_n e^{i\kappa_n a \cos(Kx)} e^{inKx}.
\]
\[ \eta_0 e^{i \kappa a \cos(Kz)} - \eta_0 \sum_{n=-\infty}^{\infty} R_n \left[ \frac{\kappa_n}{\kappa} - \frac{nK^2 a}{\kappa} \sin(Kz) \right] \]
\[ \times e^{-i \kappa a \cos(Kz)} e^{i Kz} = \xi \sum_{n=-\infty}^{\infty} T_n \left[ \frac{k_n}{k} + \frac{nK^2 a}{k} \sin(Kz) \right] \]
\[ \times e^{i k_n a \cos(Kz)} e^{i Kz}. \]

(17)

We can re-write these two boundary conditions by exploiting the previously mentioned symmetry which requires that \( R_n = R_{-n} \) and \( T_n = T_{-n} \) and the well-known Jacobi–Anger expansion
\[ e^{i \phi \cos(\theta)} = \sum_{\ell=-\infty}^{\infty} i^\ell J_\ell(\phi)e^{i \ell \phi} = \sum_{\ell=0}^\infty i^\ell q_\ell J_\ell(\phi)(e^{i \ell \phi} + e^{-i \ell \phi}), \]

(18)

where \( q_\ell = 1/2 \) if \( \ell = 0 \) and \( q_\ell = 1 \) else and \( J_\ell \) is the \( \ell \)-th order Bessel function of the first kind. After some algebra, from Eqs. (16) and (17) we get
\[ \sum_{\ell=0}^{N} q_\ell e^{i \ell J_\ell(\kappa a)} = - \sum_{n=0}^{N} R_n q_n \sum_{\ell=0}^{N} q_\ell [i^{\ell-n}J_{\ell-n}(-\kappa_0 a)] \]
\[ + i^{\ell-n}J_{\ell-n} (-\kappa_0 a)] \]
\[ \times \sum_{n=0}^{N} q_\ell [i^{\ell-n}J_{\ell-n}(k_n a) + i^{\ell+n}J_{\ell+n}(k_n a)] \]
\[ \sum_{\ell=0}^{N} q_\ell [i^{\ell-n}J_{\ell-n}(k_n a)] \]

(19)

and

\[ \sum_{\ell=0}^{N} q_\ell e^{i \ell J_\ell(\kappa a)} = \sum_{n=0}^{N} R_n q_n \sum_{\ell=0}^{N} q_\ell [i^{\ell-n}J_{\ell-n}(-\kappa_0 a)] \]
\[ + \frac{nK^2 a}{2i\kappa} \sum_{\ell=0}^{N} q_\ell [i^{\ell-n-1}J_{\ell-n-1}(-\kappa_0 a)] \]
\[ - i^{\ell+n-1}J_{\ell+n-1}(-\kappa_0 a) \]
\[ - i^{\ell-n-1}J_{\ell-n+1}(-\kappa_0 a) \]
\[ + \frac{k_n}{k} \sum_{\ell=0}^{N} q_\ell [i^{\ell-n}J_{\ell-n}(k_n a)] \]
\[ + i^{\ell+n}J_{\ell+n}(k_n a)] \]

\[ \sum_{\ell=0}^{N} q_\ell [i^{\ell-n}J_{\ell-n}(k_n a)] \]

2.3. Apparent electrical conductivity

In eddy current inspection, the electrical conductivity of the specimen is usually determined from the impedance of a probe coil by suppressing the adverse lift-off effect based on the differences between the trajectories of the lift-off and conductivity curves in the complex impedance plane [11]. The lift-off effect is caused by variations in the electromagnetic coupling between the primary probe coil and the secondary eddy current loop in the material as the finite-size probe approaches the surface of the specimen to be inspected, therefore the larger the probe, the smaller the effect of a given lift-off distance. Although the complex impedance of a finite-size probe coil over a corrugated or randomly rough half-space could be calculated using either classical analytical [12,13] or more contemporary numerical methods [14,15], the main issue of the increasing apparent loss of electrical conductivity at increasing inspection frequencies would be unnecessarily complicated by the unrelated issue of lift-off. Instead, we can simply consider an infinitely large probe coil that produces no lift-off effect at all. In that case, the primary field is a normally incident electromagnetic wave, while the only significant component of the secondary field is the specularly reflected coherent wave. Accordingly, regardless of the number of terms included in the numerical solution in order to achieve convergence, the apparent electrical conductivity of the material will be determined solely from the zeroth-order reflection coefficient \( R_0 \).

For a smooth surface (\( a = 0 \)), Eqs. (19) and (20) yield
\[ 1 = -R_0^{\text{smooth}} + T_0^{\text{smooth}} \]
\[ \text{and} \quad 1 = R_0^{\text{smooth}} + T_0^{\text{smooth}} \]
\[ \frac{\xi}{\eta_0} \text{ for } \ell = 0 \]

and
\[ 0 = -R_0^{\text{smooth}} + T_0^{\text{smooth}} \]
\[ \text{and} \quad 0 = R_0^{\text{smooth}} \frac{\kappa_0}{\kappa} + T_0^{\text{smooth}} \frac{\xi k_0}{\eta_0} \text{ for } \ell \neq 0. \]

Based on Eqs. (21) and (22), all the higher-order (\( \ell \neq 0 \)) reflection and transmission coefficients are identically zero, while the sought zeroth-order reflection coefficient can be calculated as follows
\[ R_0^{\text{smooth}} = \frac{1 - \frac{\xi}{\eta_0}}{1 + \frac{\xi}{\eta_0}} \approx 1 - 2 \frac{\xi}{\eta_0}. \]

(23)

Generally, the measured complex reflection coefficient from a rough surface can be written as
\[ R_0 = [1 - \alpha(1 - i)] e^{i \beta}, \]
\[ \text{where } \alpha \text{ and } \beta \text{ represent the effects of the surface irregularity on the reflection measurement:} \]
\[ \alpha = \frac{2 \omega \mu \varepsilon_0}{\mu_0 \sigma_0} \quad \text{and} \quad \beta = 2 \ell \kappa, \]
\[ \text{respectively. Eqs. (19) and (20) represent } 2N + 2 \text{ equations for } 2N + 2 \text{ unknowns (} R_0, R_1, ..., R_N, T_0, T_1, ..., T_N) \text{, which can be readily solved by numerical means.} \]
and they are both much smaller than unity. Here, $\sigma_0$ and $\ell_a$ denote the ‘apparent’ electrical conductivity and lift-off distance, respectively. For simplicity, these parameters can be normalized to the actual conductivity $\sigma$ of the material and to the amplitude of the surface corrugation $a$ as $\sigma_a = \gamma \sigma$ and $\ell_a = \xi_a$, respectively, where $\gamma$ and $\xi$ are the normalized apparent conductivity and the normalized lift-off distance. The separation of variables represented by Eq. (24) is analogous to the separation of lift-off and conductivity effects in the complex impedance of a probe coil [11]. An example of the practical realization of this method is described in Ref. [5] in connection with the experimental investigation of the apparent conductivity loss in shot-peened copper specimens.

For small perturbations, Eq. (24) can be approximated by

$$ R_0 \approx (1 - \alpha) + i(\alpha + \beta), $$

so that

$$ \gamma \approx \frac{2 \omega \mu_0 \varepsilon_0}{\sigma \mu_0 (\text{Re}\{R_0\} - 1)^2} $$

and

$$ \xi = \frac{\text{Im}\{R_0\} + \text{Re}\{R_0\} - 1}{2\kappa a}. $$

### 3. Numerical examples

In this section we present a few numerical examples to illustrate the apparent conductivity loss on rough surfaces for one-dimensional sinusoidal corrugation in the case of transverse magnetic (TM) polarization, i.e. when the eddy current flows normal to the periodic grooves. The relevant physical parameters and material properties used in the calculation are as follows: the amplitude of the surface corrugation ($a$), the period of the corrugation ($\Lambda$), the inspection frequency ($f$), the permittivity ($\varepsilon_0 = 8.854 \times 10^{-12}$ As/Vm) and permeability of a vacuum ($\mu_0 = 4\pi \times 10^{-7}$ Vs/Am), the electrical conductivity of the conductor ($\sigma = 58 \times 10^7$ A/Vm for copper), and the permeability of the conductor ($\mu = \mu_0$ for copper).

As an example, Fig. 2(a) shows the variation of the normalized conductivity with frequency for a given amplitude-to-period ratio of $a/\Lambda$ = 0.05. As expected, the measured conductivity decreases with increasing inspection frequency. This is in accordance with the earlier explanation that at higher frequencies the eddy current flows closer to the surface, therefore follows a more tortuous path, which translates into an apparent loss of electrical conductivity. The maximum loss of conductivity at high frequencies is related to the ‘sharpness’ of the surface irregularity. For a randomly rough surface, this sharpness is determined by the ratio of the RMS roughness $h$ and the correlation length $L$ of the surface. For the simple sinusoidal corrugation considered here, the sharpness can be characterized by the correlation length $L$.

The maximum loss of conductivity at high frequencies is significant at high frequencies, but entirely diminishes at low frequencies. What happens is that at low frequencies the surface corrugation forces an asymmetric distribution on the eddy current density with respect to the average plane ($z = 0$). This asymmetric current distribution exhibits a small shift of the average towards positive values of $z$, which is equivalent to a small constant lift-off.

In comparison, at high frequencies the eddy current follows the sinusoidal profile of the surface, therefore the lift-off distance decreases with increasing frequency, although the overall change is less than 15% of the corrugation amplitude, which is clearly negligible in practice. Still, it is interesting that the apparent lift-off caused by the perturbation of the surface is perceivable at low frequencies, but entirely diminishes at high frequencies. In spite of the similarity of the conductivity and lift-off curves shown in Fig. 2, this frequency dependence is essentially the opposite of the behavior exhibited by the apparent conductivity loss, which is significant at high frequencies, but entirely diminishes at low frequencies. What happens is that at low frequencies the surface corrugation forces an asymmetric distribution on the eddy current density with respect to the average plane ($z = 0$).
the average depth of the current is not affected by the corrugation and the lift-off diminishes. However, this effect is negligible with respect to the actual lift-off encountered in real inspection, therefore will be disregarded in the rest of this paper.

3.1. Numerical convergence of the Rayleigh–Fourier method

The $2N + 2$ linear equations of Eqs. (19) and (20) can be readily solved for $2N + 2$ unknowns ($R_0, R_1, \ldots, R_N, T_0, T_1, \ldots, T_N$) for an arbitrary set of parameters. Then, the normalized apparent electrical conductivity of the material can be determined from the zeroth-order reflection coefficient ($R_0$) using Eq. (27). In all the calculations presented elsewhere in this paper, the infinite series in the ‘exact’ solution was truncated at $N = 9$, which limited the dimension of the largest matrix to be inverted to 20-by-20. In order to verify that the above described plateau region is not caused by numerical artifacts, additional runs were made using different values of $N$. Fig. 3(a) shows results obtained for $a = 50 \mu m$ and $\Lambda = 1000 \mu m$ at different truncation values. The results are obviously highly convergent and only the lowest truncation level ($N = 3$) is distinguishable at this scale from the rest of the curves ($N = 9, 7, 5$) above 10 MHz. In order to further illustrate the excellent convergence of the method, Fig. 3(b) shows the difference between curves of different truncation levels ($N = 3, 4, 5, and 7$) and the one corresponding to the highest truncation level ($N = 9$), which was used as a reference. At this enlarged scale, the numerical error is more visible and we can establish that the error is less than $10^{-5}$ up to 100 MHz, which is quite negligible.

3.2. High-frequency plateau, Kirchhoff approximation

Our previous results indicated that at high frequencies the apparent conductivity approaches a constant asymptotic value $\gamma_\infty$, that is solely determined by the $a/\Lambda$ ratio. Fig. 4 shows this high-frequency plateau region for four different values of $a$ at $\Lambda = 1000 \mu m$. The transition frequency is determined by the period of the surface corrugation. Above this frequency, the penetration depth of the eddy current becomes negligible with respect to the period of the surface and the apparent loss of conductivity can be approximated as a simple geometrical effect due to the tortuous path followed by the eddy current. This high-frequency asymptotic behavior of the apparent conductivity can be readily investigated using physical approximations that allow us to derive an explicit analytical formula for $\gamma_\infty$, which can then be compared to our numerical results. According to the so-called Kirchhoff approximation, at any point sufficiently close to the surface the wave field can be treated as if the surface were part of an infinite tangential plane and the scattered field at some distance from the surface can be obtained by integration over all the points on the surface.

Fig. 5 shows a schematic diagram of the incident ray at point $P(x)$ hitting the corrugated surface previously defined by Eq. (1) at an angle of incidence of

$$\theta_i(x) \approx \frac{dx}{d\nu} = -\frac{2\pi a}{\Lambda} \sin\left(\frac{2\pi x}{\Lambda}\right).$$

When considering the resulting eddy current distribution in the conducting substrate, phase differences in the incident field at the surface can be neglected since $\lambda \gg a$. 

![Fig. 3](image1.png)
![Fig. 4](image2.png)
The electrical field along the tangential plane will be simply reduced by a factor of \( \cos \theta_i \) with respect to the smooth surface, therefore the eddy current density will also proportionally decrease. Otherwise, the depth distribution of the eddy current will be the same. The zeroth-order reflected field will be proportional to the average of the \( x \)-component of the eddy current field, therefore the normalized apparent conductivity will be simply

\[
\gamma_{ao} = 1 - \frac{4\pi^2 \alpha^2}{\Lambda^2} \left( \sin^2 \left( \frac{2\pi}{\Lambda} x \right) \right) = 1 - \frac{2\pi^2 \alpha^2}{\Lambda^2}.
\]  

(29)

Fig. 6 shows the high-frequency asymptotic value \( \gamma_{ao} \) of the normalized apparent conductivity for three different values of the surface period \( \Lambda \) as a function of the surface amplitude \( \alpha \). The solid lines were calculated from the numerical solution at a very high frequency \( (f = 100 \text{ MHz}) \) while the dashed lines were calculated from the approximate solution of Eq. (29). The agreement between the numerical solution and the asymptotic Kirchhoff approximation is excellent whenever the \( \alpha/\Lambda \) ratio is sufficiently small.

\[
\gamma(f) \approx \frac{1 + \pi f_2}{1 + \pi f_1},
\]  

(30)

where \( f_1 \) and \( f_2 \) are characteristic frequencies that depend on the surface parameters and the material properties. At very low and very high frequencies, the approximate formula of Eq. (30) approaches unity and \( \gamma_{ao} = f_1/f_2 \), respectively, where \( \gamma_{ao} \) can be calculated from Eq. (29). For slightly rough surfaces the two characteristic frequencies are very close to each other, i.e. \( f_1 \approx f_2 \). The transition from low-frequency to high-frequency behavior occurs when \( \Lambda/\delta \approx \pi \), where \( \delta = \sqrt[4]{\mu \sigma / \pi} \) is the standard penetration depth of the eddy current distribution. Therefore, we can calculate \( f_1 \) from

\[
f_1 = \frac{\pi}{\mu \sigma \Lambda^2},
\]  

(31)

and subsequently \( f_2 \) from

\[
f_2 = \frac{f_1}{1 - 2\pi^2 \alpha^2 / \Lambda^2}.
\]  

(32)

Fig. 7 shows a comparison between the numerical solution (solid lines) and the proposed approximate formula (dashed lines) for four different values of \( \alpha \) at \( \Lambda = 1000 \mu m \). As expected, the high-frequency conductivity loss is somewhat

3.3. Explicit ad hoc approximation

From a practical point of view, it would be very beneficial if we could directly predict the surface roughness induced apparent loss of eddy current conductivity from surface parameters that can be readily measured by optical or other means. In the case of a corrugated surface of sinusoidal profile these parameters are the surface amplitude \( \alpha \) and the surface period \( \Lambda \). By inspection of the previously presented numerical results, we found that the functional dependence of the normalized apparent conductivity can be approximated by the following ad hoc function

\[
\gamma(f) \approx \frac{1 + \pi f_2}{1 + \pi f_1},
\]  

(30)

Fig. 5. A schematic diagram of an incident ray hitting the corrugated surface.

Fig. 6. The high-frequency asymptotic value \( \gamma_{ao} \) of the normalized apparent conductivity for three different values of the surface period \( \Lambda \) as a function of the surface amplitude \( \alpha \). The solid lines were calculated from the numerical solution at a very high frequency \( (f = 100 \text{ MHz}) \) while the dashed lines were calculated from the approximate solution of Eq. (29).

Fig. 7. Comparison between the numerical solution (solid lines) and the proposed approximate formula (dashed lines) for four different values of \( \alpha \) at \( \Lambda = 1000 \mu m \). As expected, the high-frequency conductivity loss is somewhat...
overestimated by Eq. (29) for large $a/L$ ratios, but otherwise the agreement is fairly good. Considering the simplicity of the approximate ad hoc formula and the expected uncertainties in the measurements of the eddy current conductivity and the surface parameters, the illustrated accuracy is quite sufficient for predicting the expected apparent electrical conductivity loss in most practical situations.

4. Comparison to experimental data from randomly rough surfaces

The limited objective of this paper has been to model a one-dimensional rough surface of simple sinusoidal profile using the Rayleigh–Fourier method in order to investigate the previously observed apparent loss of conductivity. For two-dimensional randomly rough surfaces our analysis can be extended by applying Rice’s method which models the rough surface using appropriately chosen random coefficients for its 2D Fourier expansion [16]. This method could provide a more general model that would be quantitatively accurate and practically applicable to the development of corrections for eddy current conductivity measurements on rough surfaces. Since the preliminary results presented in this paper apply to sinusoidally corrugated surfaces only, they do not really give true quantitative predictions for randomly rough surfaces. Still, it is interesting to compare the predictions of this simple approximation to previously published experimental data [5] and show that the somewhat oversimplifying sinusoidal model still qualitatively predicts both the absolute value and the frequency dependence of the apparent conductivity loss.

Fig. 8 shows the experimentally determined RMS roughness and correlation length in shot-peened pure copper specimens of different Almen intensities. In order to better capture the dominant statistical features of the data, the experimental results were best fit with simplified trend lines shown as solid lines. Based on these experimental results, in the following illustration the RMS roughness $h$ and correlation length $L$ of the randomly rough surface will be approximated as

$$
\begin{align*}
    h &\approx 23.5 - \frac{22}{1 + 0.063A} \\
    L &\approx 157 - \frac{122}{1 + 0.39A},
\end{align*}
$$

(33)

where $A$ denotes the shot peening Almen intensity. The correlation function of the two-dimensional random roughness on these surfaces was found to be well approximated by a logarithmic distribution. Naturally, such surfaces cannot be accurately modeled by a one-dimensional regular corrugation of sinusoidal profile, which was considered in our analysis. Still, the consistency of these predictions with the measured loss of conductivity can be readily checked by establishing a crude correspondence between the amplitude and period of the equivalent sinusoidal corruga-
tion on one side and the RMS roughness and correlation length of the random surface roughness on the other side. A brief statistical analysis revealed that relatively good overall correspondence can be achieved by choosing

$$
\begin{align*}
    a &\approx 3.3h \\
    A &\approx 2.5L.
\end{align*}
$$

(34)

Fig. 9 shows the comparison between the experimentally determined normalized conductivity (symbols) and the approximate solution (solid lines) for shot-peened pure copper specimens using the regression procedure of Eqs. (33) and (34). In spite of the very crude nature of the analytical model and the inherent experimental uncertainties caused partly by measurement errors and partly by the limited number and size of the specimens (a single $1\times 1$ shot-peened area for each Almen intensity), there is an obvious overall consistency between the measured and predicted data. However, further analytical efforts are needed to properly account for the true random nature of the surface roughness produced by shot peening and
a larger, statistically more representative, set of specimens must be inspected before a sufficiently accurate surface roughness correction technique can be developed for high-precision eddy current conductivity measurements on surface-treated metals.

5. Conclusions

The apparent loss observed in eddy current electrical conductivity measurements on specimens of rough surfaces can lead to significant inaccuracy in the measured values and in extreme cases might completely overshadow the weak variation in the sought intrinsic material property. In order to better understand and potentially correct for this spurious effect, a simple analytical model has been proposed based on the assumption that randomly rough surfaces can be crudely approximated by a corrugated surface of sinusoidal profile. The analytical technique used to calculate the reflection coefficient from the corrugated surface of a conducting half-space is based on the Rayleigh–Fourier method. Although the Rayleigh–Fourier method is well established in the scientific literature for such calculations, to the best of our knowledge, there has not been an effort until now to exploit it for studying the apparent conductivity loss observed in eddy current materials characterization.

We also derived an explicit high-frequency asymptotic expression for the reduced conductivity using the Kirchhoff approximation and demonstrated that for slight corrugation the numerical solution based on the Rayleigh–Fourier method converges to the Kirchhoff approximation. The additional physical insight provided by the Kirchhoff approximation led us to develop a very simple explicit ad hoc expression that well approximates the numerical predictions of the Rayleigh–Fourier method over the whole frequency range. This explicit formula can be used as a practical tool to directly estimate the apparent conductivity loss for a sinusoidal corrugation of known amplitude and period and could prove to be very useful in increasing the accuracy of the measured conductivity values in experimental studies. Finally, we compared our analytical results to previously published experimental data and showed that the sinusoidal model qualitatively predicts both the absolute value and the frequency dependence of the apparent conductivity loss. This comparison also indicates the need for future research efforts to further increase the quantitative accuracy of the analytical model. This can be achieved by extending our present technique using Rice’s method, which is based on a two-dimensional Fourier decomposition of the random surface height distribution. In summary, the presented numerical technique and the approximate explicit ad hoc formula allow the surface roughness effects to be better isolated from true electrical conductivity variations in eddy current measurements thereby increasing the accuracy of near-surface materials characterization in surface-treated or otherwise roughened specimens.

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