Surface roughness induced attenuation of reflected and transmitted ultrasonic waves

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The problem of ultrasonic transmission and reflection at a randomly rough interface is considered in connection with ultrasonic NDE of rough surface samples by immersion method. A simple first-order phase perturbation technique is used to calculate both transmitted and reflected components for comparison with experimental results. The transmitted wave is shown to be attenuated in a similar way to the reflected one, and their attenuation ratio is found to be independent of frequency in the considered cases of slight surface roughness. For instance, the surface roughness induced attenuation of the wave reflected from a water–aluminum interface is about seven times higher than that of the transmitted component. Experimental results are presented to show good agreement with calculated predictions of the suggested simple technique.

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INTRODUCTION

The interaction of an acoustic wave with a randomly rough boundary separating two different media has been studied for a long time and continues to be of considerable interest. The difficulty of the problem lies in the complexity of the generated fields: A single incident wave can produce both longitudinal and shear bulk waves in both media, and surface waves along the interface as well. Of course, all of these waves will be more or less diffuse scattered fields depending on the surface roughness. Practically all theoretical1–5 and experimental6–9 works in this field deal with the reflected part, i.e., with the distribution of the acoustic energy scattered back into the medium of incidence. On the other hand, much less is known about the transmitted part, i.e., the scattered field in the second medium. Although this field usually contains only a negligible part of the incident energy, it becomes of primary importance whenever a solid sample of rough surface is interrogated by ultrasonic immersion technique.

Another related field which shares the interest in acoustical transmission through randomly rough boundaries is oceanographic acoustics. The effect of a random ocean surface on a low-frequency acoustic signal emanating from a point source in the water was studied in detail.10,11 These results cannot be easily adapted to our much simpler case mainly because we deal with plane waves (rather than point sources) in the ultrasonic range.

I. SURFACE ROUGHNESS INDUCED ATTENUATION

In the following, we shall introduce a very simple technique to account for the effect of slight surface roughness on the transmission of a well-collimated ultrasonic beam through an otherwise flat interface at normal incidence. Let us have a randomly rough liquid–solid interface \( h(x,y) \) positioned in the \( z = 0 \) plane of an \( x,y,z \) coordinate system, as is shown in Fig. 1.

The rough interface under study is supposed to be geometrically flat over the insonified area \( A \):

\[
\int_A \int h(x,y) \, dx \, dy = 0,
\]

and the surface quality is characterized by a single effective (rms) roughness parameter \( h \):

\[
h^2 = \frac{1}{A} \int_A \int h^2(x,y) \, dx \, dy.
\]

The effect of surface roughness is regarded as a weak perturbation of the known plane-wave solution involving the same refractive indexes and a smooth surface. This approximation is limited to slightly rough surfaces when \( h \) is small with respect to both the acoustic wavelength and the correlation length of the roughness; i.e., the average curvature of the surface is small.

Without surface roughness, the reflected and transmitted fields would be simple plane waves propagating in opposite directions. Let \( R_0 \) and \( T_0 \) denote the well-known reflection and transmission coefficients for normal incidence on a smooth plane interface. In the presence of surface roughness, both reflected and transmitted waves become complex scattered fields. We are interested in the modified plane-wave reflection \( R \) and transmission \( T \) coefficients, so we shall completely disregard the so-called incoherent components of the scattered fields and base our calculations simply on the reduced strength of the coherently scattered “specular” components. This truly “plane” wave approach sets yet another restriction on the feasibility of the resulting theoretical predictions. The diameter of the well-collimated beam should be large enough so that the divergence of the specular components is small with respect to that of the neglected scattered waves. We shall presume that the incident energy
is divided into reflected and transmitted parts in the same way as in the case of a smooth plane surface, but these components are perturbed by \( \phi_r(x,y) \) and \( \phi_s(x,y) \) random phase modulations, respectively. The resulting plane-wave attenuation can be derived directly from the angular frequency representation of the perturbed specular field. The coherent specular wave\(^{12,13}\) is given as

\[
E = \frac{E_0}{A} \int \int e^{i\phi(x,y)} \, dx \, dy, \tag{3}
\]

where \( E_0 \) is the complex amplitude of the undisturbed field without surface roughness and \( \phi(x,y) \) is the random phase modulation due to the surface roughness.

According to the phase perturbation approximation, the rough surface acts like a thin phase plate. The surface roughness induced phase modulations can be easily expressed by \( h(x,y) \):

\[
\phi_r(x,y) = -2h(x,y)k_1, \tag{4}
\]

and

\[
\phi_s(x,y) = -h(x,y)(k_1 - k_2), \tag{5}
\]

where \( k_1 \) and \( k_2 \) denote the longitudinal wavenumbers in the first and the second media, respectively. Here, we presumed that the correlation length of the rough surface is high with respect to the wavelength (Kirchhoff approximation). Therefore, the following results are expected to yield reliable predictions in the overlapping domain of the “small-amplitude, small-slope” phase perturbation and the Kirchhoff approximations only.\(^{14}\) Presuming that the surface profile is an ergodic random process, we can conclude from Eqs. (4) and (5) that the phase perturbation is ergodic, too. Under these conditions, the area integral of Eq. (3) can be expressed by the probability density distribution \( p(\phi) \) of the random phase modulation:

\[
E = E_0 \int_{-\infty}^{\infty} e^{i\phi} p(\phi) \, d\phi. \tag{6}
\]

Equations (3) and (6) are well known for determining the coherent specular part of the reflected field, but, to the knowledge of the authors, they have never been applied to the transmission problem. For later comparison with the experimental results, let us solve Eq. (6) for the Gaussian distribution, when the probability density function is

\[
p(\phi) = \frac{1}{\sqrt{2\pi}\phi_s} e^{-\phi^2/2\phi_s^2}, \tag{7}
\]

where \( \phi_s \) is the rms value, or spread of the distribution. Substituting Eq. (7) into Eq. (6) gives

\[
E = \frac{E_0}{\sqrt{2\pi}\phi_s} \int_{-\infty}^{\infty} e^{i\phi} e^{-\phi^2/2\phi_s^2} \, d\phi. \tag{8}
\]

The solution of Eq. (8) is well known:

\[
E = E_0 e^{-\phi_s^2/2}. \tag{9}
\]

Figure 2 shows the phase perturbation induced attenuation of the specular component for five different probability distributions of practical importance.\(^{12,13}\) Whenever the surface roughness induced attenuation is less than 3 dB, the rms roughness more or less uniquely determines the result. In the case of stronger phase modulation, the widely used Gaussian distribution yields the smallest attenuation while other distributions result in more serious reduction of the specular component.

Finally, the modified reflection and transmission coefficients for Gaussian distribution can be written from Eq. (9) as follows:

\[
R = R_0 e^{-2k_1^2}, \tag{10}
\]

and

\[
T = T_0 e^{-1/2k_1^2(k_1 - k_2)^2}. \tag{11}
\]

Equation (10) is the well-known formula often used in

**FIG. 2.** Surface roughness induced attenuation versus rms phase modulation for different surface profiles (1—Gaussian, 2—Simpson, 3—triangular, 4—sinusoidal, 5—rectangular).
ultrasonic surface roughness studies, but Eq. (11) gives us the sought tool to handle transmission as well.

II. EXPERIMENTAL RESULTS

The following experimental results were obtained from sand blasted aluminum surfaces. We checked the surface by a mechanical profilometer and found its distribution to be close to Gaussian. Figure 3 shows the results of the profilometer measurement of 550 points over a 30-× 30-mm area as well as the best fitting Gaussian curve of 14-μm rms roughness. A more detailed former study of similar surfaces showed that the coherence length of the surface profile increased, but very slowly with \( h \) in the 6- to 90-μm rms range. The coherence length rms roughness ratio was found to be at least 10 below \( h = 20 \mu m \); therefore, we chose our samples in this range so that we could compare our experimental results to the predictions of the above introduced analytical model. According to Fig. 3, the probability density distribution of the sand blasted aluminum surface can be approximated by Gaussian distribution; therefore, when comparing our experimental results to theoretical predictions, we shall always use this distribution.

Figure 4 shows the schematic diagram of the experimental arrangement. Only the front-wall echo and the first back-wall echo were spectrum analyzed. At first we measured these spectra at ten positions over a 50-× 50-mm area, and then we averaged the power densities in order to smoothen the resulting surface roughness attenuation curves. The averaged spectra were compared to the front- and back-wall spectra from a smooth sample of the same size and material, and positioned at the same distance from the transducer. In this way, the reduced strength of the rough surface signals is entirely due to the sought scattering induced attenuation at the boundaries of the sample. The total attenuation of the back-wall echo includes all three interactions with the rough surface, i.e., reflection from the aluminum–water interface and two transmissions through it (the transmission loss is the same in both directions).

The 12.5-mm-diam transducer insonified the 25-mm-thick samples from 100 mm at normal incidence. Of course, a finite beam transducer is somewhat sensitive to the incoherent scattered field as well; however, we found that this effect was negligible in our experiments. The detected time signal itself would readily show the presence of unwanted incoherent components as spatial noise or speckle. Furthermore, we checked that the received signal dropped dramatically when we slightly misaligned the incident angle, which indicates that the specular component is much stronger than the diffuse field.

Figure 5 shows the experimental results and theoretical curves calculated by Eq. (10) for two different surface

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FIG. 3. Experimental density distribution with the best fitting Gaussian curve for sand blasted aluminum surface of \( h = 14 \mu m \).

FIG. 4. Schematic diagram of the experimental setup.
roughnesses. The good quantitative agreement between the experimental and theoretical data shows that the surface roughness induced attenuation of the specularly reflected signal is a very good indication of the surface quality, a fact which can be taken advantage of for rms surface roughness measurements.

The real advantage of the phase modulation approach is its ability to describe the transmission as well. According to this very simple analysis, the surface roughness attenuates the reflected and transmitted components in a similar way, and their attenuation ratio is independent of frequency. Figure 6 shows the calculated reflection and transmission attenuation coefficients for a water–aluminum interface. In addition, the attenuation coefficients of the through-transmitted and back-wall signals for a sample with rough front and back surfaces of 14-μm rms are shown, which are of practical importance in immersion applications.

It is very important to recognize that, due to the substantial velocity difference between water and aluminum, the transmitted wave is much less affected by the surface roughness than the front-wall reflection. Furthermore, the through-transmitted and back-wall reflected signals are less attenuated than the front-wall reflection as well, in spite of their multiple interaction with the attenuating rough surface. This somewhat unexpected phenomenon is shown in Fig. 7, where we compared theoretical and experimental results for an aluminum sample of 18-μm rms roughness.

III. CONCLUSIONS

A simple analytical method based on the phase perturbation of the acoustical field was introduced to calculate the surface roughness induced attenuation of the reflected and transmitted specular waves at normal incidence. Only the coherently reflected and transmitted wave components were regarded, but the resulting surface roughness attenuation values are valid approximately for any well-collimated finite beams as well, when the beam diameter is much higher than both the wavelength and the correlation length of the surface roughness. Without this restriction, the received signal inevitably includes some incoherently scattered components, too, and the surface roughness induced attenuation becomes somewhat smaller.

The surface roughness induced attenuation mainly depends on the rms roughness, but, in case of strong roughness, it becomes increasingly dependent on the surface profile as
well. In agreement with the generally accepted assumption, sand blasted aluminum surfaces were found to have approximately Gaussian density distribution.

In the case of water–aluminum interface, the transmitted wave is much less attenuated than the reflected one, and somewhat more than the reflection in the opposite direction. This will result in an interesting phenomenon: The surface roughness induced attenuation of the whole sample seems to be negative, causing underestimation of the volume induced attenuation of the interrogating wave. The surface roughness induced attenuation calculated by the suggested simple model was found to be in good agreement with experimental results for both reflected and transmitted cases.

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