Viscosity-induced attenuation of longitudinal guided waves in fluid-loaded rods

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The main goal of this paper is to extend the analytical treatment of longitudinal wave propagation along the fiber direction of multilayered coaxial fibers to immersion in a viscous fluid. The viscous fluid is modeled as a hypothetical isotropic solid having rigidity \( c_{44} = -i \omega \eta \), where \( \eta \) denotes the viscosity of the fluid and \( \omega \) is the angular frequency, i.e., the vorticity mode associated with the viscosity of the fluid is formally described as the shear mode in the hypothetical solid. Among other interesting phenomena, the analytical results revealed the presence of a sharp minimum in the viscosity-induced attenuation of the lowest-order longitudinal mode of thin rods. This minimum occurs at a particular frequency when the otherwise elliptical polarization of the surface vibration becomes linearly polarized in the radial direction. Generally, the experimental results on immersed and fluid-covered wires and fibers showed good agreement with the analytical predictions. In particular, the existence of the theoretically predicted minimum in the attenuation spectrum of the lowest-order longitudinal mode was verified. © 1996 Acoustical Society of America.

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INTRODUCTION

There are three types of modes that can propagate along a cylindrical rod: axially symmetric longitudinal waves which involve both radial and axial displacements, torsional waves which involve circumferential displacement only, and flexural modes which involve radial, axial, and circumferential displacements and do depend on the azimuthal angle.\(^1\) Longitudinal guided wave propagation along cylindrical rods is of great practical importance in numerous areas of acoustics. For example, such guided modes are often used to evaluate the material properties of thin metal wires, optical fibers, and reinforcement filaments used in epoxy, metal, and ceramic matrix composites. Generally, the intrinsic properties of single or coaxially layered composite rods are most easily evaluated from dispersion measurements made on the free rod. However, immersed rods can also be used to measure rod properties such as Poisson’s ratio which are strongly related to the fluid loading on the rod.\(^2\) Guided waves in embedded fibers were recently studied in great detail by Simmons et al.\(^3,4\) The feasibility of using such longitudinal guided waves to evaluate the interface properties between the fiber and the surrounding solid matrix was also investigated by both theoretical and experimental means.\(^5\) Another interesting application is when an embedded fiber is used to monitor the properties of the surrounding material during polymerization.\(^6\) In this case, the fiber properties are known and the interface conditions are assumed to be perfect. Any change in the velocity or attenuation of the guided mode propagating in the fiber can then be attributed to the surrounding material that changes from a viscous fluid to an elastic solid during the curing process to be monitored. Guided modes producing mainly tangential surface displacements can be readily used to measure fluid viscosity. From this point of view, torsional modes are the best,\(^7,8\) but they are more difficult to generate and detect than extensional modes.\(^9\)

Most of the theoretical works in this area considered clad rods consisting of an isotropic core and an arbitrary number of isotropic coatings. A comprehensive review of these studies was published by Thurston.\(^1\) More recent efforts further developed this microscopically isotropic approach by including the effects of imperfect interfaces between the coaxial layers.\(^10\) At sufficiently low frequencies, such clad rods might appear strongly anisotropic with one axis of symmetry along the geometrical axis of the rod. Much less attention was paid to the case of anisotropic rods. Relatively few efforts have been spent upon the study of axisymmetric guided wave propagation in free and immersed anisotropic rods of transversely isotropic nature.\(^11-16\)

These works are insufficient to model many situations encountered in materials characterization of advanced fiber-reinforced composites. In order to better model acoustic wave propagation in such materials at least three major effects need to be accounted for. These are the inhomogeneous nature of the structure as reflected in its multilayering, the inherent microscopic anisotropy of some of the constituents, and the quality of the interfaces. In a recent paper, we described a unified analytical treatment of wave propagation along the axial direction of multilayered anisotropic coaxial fibers.\(^17\) This approach can be used to study longitudinal guided wave propagation along a multilayered fiber that is either free, immersed in a viscosity-free fluid, or embedded in an elastic-solid. Each of the involved material components are allowed to possess transverse or complete isotropy that insures axial symmetry along the fibers. The model can be also used to account for the presence of imperfect interface zones. The interface zone can be treated as a thin layer of degraded mechanical properties or by introducing a finite
interfacial stiffness condition. Figure 1 shows the schematic diagram of the coaxially layered structure and the numbering strategy. The analytical technique developed in our previous paper is analogous to the transfer matrix technique introduced originally for plane interfaces by Thomson and somewhat later on by Haskell that has been used extensively ever since in a wide variety of applications.

I. VISCOS FLUID LOADING

In spite of the very general nature of our recently developed analytical technique, it did not intend to model one very important class of wave propagation phenomena, namely the excess loss of guided waves propagating in rods that are either immersed in or coated with a viscous fluid. The main goal of this paper is to extend our previous analytical model to viscous fluids of the Newtonian type that can be characterized by a single frequency-independent viscosity parameter. A fiber immersed in such a viscous fluid can be simply modeled as a fiber embedded in a hypothetical isotropic solid having rigidity \( c_{44} = -i \omega \eta \), where \( \eta \) denotes the shear viscosity of the fluid (see, e.g., Ref. 20). In this way, the vorticity mode associated with the viscosity of the fluid is formally described as the shear mode in the hypothetical solid. This hypothetical solid is isotropic, therefore its three different elastic constants are related via \( c_{11} = 2c_{44} \). Within the limitation of this relationship, \( c_{11} \) and \( c_{12} \) must be chosen in accordance with the physical behavior of the viscous fluid. For this purpose, we adopted the so-called Stokes model for viscous fluids by choosing \( c_{11} = \lambda + 4/3c_{44} \) and \( c_{12} = \lambda - 2/3c_{44} \), where \( \lambda \) denotes the compressibility of the viscosity-free fluid. The same method was used recently by Liu et al. to study the behavior of extensional modes in a thin rod immersed in a viscous fluid with the exception that they also incorporated the usually negligible second or volume viscosity in \( c_{11} \). The primary effect of viscosity is the appearance of the vorticity mode; however, it also causes a small attenuation of the longitudinal "acoustic" mode. This attenuation is equal to the imaginary part of the complex longitudinal wave number, i.e., \( \alpha_L = \text{Im} \left( \omega \sqrt{\frac{\rho_f}{\lambda - 4i\omega \eta/3}} \right) \), where \( \rho_f \) is the density of the fluid. According to the Stokes model, \( \alpha_L = \text{Im} \left( \omega \sqrt{\frac{\rho_f}{\lambda - 4i\omega \eta/3}} \right) \approx \frac{2\omega^2 \eta}{3c_f^2 \rho_f}, \) (1)

where \( c_f = \sqrt{\lambda/\rho_f} \) denotes the sound velocity in the viscosity-free fluid.

A similar, but somewhat less rigorous, method of accounting for the viscosity of the fluid has been recently shown by Wu and Zhu to yield the same dispersion relationship for leaky Rayleigh waves propagating along a solid/viscous fluid interface as the much more complicated solution of Qi if heat conduction is neglected. More recently, Zhu and Wu extended their approach to leaky Lamb waves propagating along a solid plate in contact with a viscous fluid.

Our analytical model allows us to numerically determine the frequency-dependent velocity and attenuation of longitudinal guided waves propagating in composite rods either immersed in or coated by a viscous fluid. In spite of all the advantages of numerically determined but otherwise "exact" solutions, wave propagation phenomena can be often better understood from simpler-approximations that capture the dominating physical mechanism in a given region. In order to gain better insight into the viscosity-induced attenuation phenomenon, we shall also introduce three such approximations. At low frequencies, separate asymptotic approximations can be obtained for thick (immersed) and thin fluid coatings. At high frequencies, the guided mode in the rod asymptotically approaches the leaky Rayleigh mode propagating on a fluid-loaded plane surface.

A. Immersed rod at low frequency

The total attenuation of the lowest-order axisymmetric longitudinal mode in a thin fiber due to fluid loading can be separated into two components. First, the normal component of the surface vibration causes losses via energy leakage into the fluid. Second, the tangential component of the surface vibration causes losses via viscous dissipation in the fluid. These two attenuation components were recently calculated for homogeneous and isotropic materials by Nagy and Kent. A brief summary of their analysis is repeated here so that it could be subsequently adapted to the case of thin fluid coatings.

Generally, the attenuation caused by viscous losses can be calculated from

\[ \alpha_v = P_v/2P, \]

(2)

where \( P_v \) denotes the power lost via viscous dissipation over a unit length of the fiber and \( P \) is the total power propagating along the fiber. Assuming that the radius of the rod is much larger than the viscous skin depth, i.e., \( a \gg \delta = \sqrt{2\pi \omega \rho_f} \), and that there is only pure axial motion in the thin fiber at low frequencies, the shear drag in the fluid is \( \tau = \eta \nu_v \delta r \), where \( r \) denotes the radial distance in the fluid from the surface (for brevity, the shear drag \( \sigma_x \) and the axial velocity \( \nu_v \) are denoted simply by \( \tau \) and \( \nu \), respectively). Substituting this constitutive relationship into the momentum equation \( \partial \nu/\partial t = \rho_f \partial v/\partial t \) yields the wave equation in the form of \( \partial^2 v/\partial r^2 = (\rho_f/\eta) \partial v/\partial t \), where \( \rho_f \) denotes the density
of the fluid. The general solution of this differential equation is

\[ \nu = \nu_1 e^{ikr} + \nu_2 e^{-ikr}, \]  

(3)

where \( k^2 = i \omega \rho \sigma \eta \) or \( k = 1/\delta + i/\delta \) and the \( e^{-i\delta} \) term was suppressed. For an infinite fluid bath, \( \nu_1 \) of Eq. (3) must vanish since there is no reflecting interface that could produce a converging wave and \( \nu_1 = 0 \), where \( \nu_1 \) denotes the tangential surface velocity of the rod, which is the same as the uniform axial velocity inside the rod. The shear drag can be calculated as

\[ \tau = -i k \eta \nu_0 e^{ikr} = \tau_0 e^{ikr}, \]  

(4)

where \( \tau_0 = \eta \nu_0 (\delta^2 - 1/\delta) \). The dissipated power over a unit surface area is

\[ I_s = \frac{1}{2} \Re[-\tau_0] \nu_0 \]  

(5)

so that the total dissipated power over a unit length of the rod is

\[ P_v = 2 \pi a I_s = \pi a \nu_0^2 (\eta \rho f \omega^2 / 2)^{1/2}. \]  

(6)

In the low-frequency approximation, the vibration amplitude is constant throughout the cross section of rod, and the total power propagating along the rod can be expressed as

\[ P_r = \frac{1}{2} \nu_0^2 (E \rho) \tau_0^{1/2} \rho^2 \pi, \]  

(7)

where \( E \) and \( \rho \) denote Young’s modulus and the density in the solid, respectively. Substituting these values into Eq. (2) yields

\[ \alpha_v = \frac{1}{a} \sqrt{\frac{\omega \eta \rho f}{2 E \rho}}, \]  

(8)

B. Thin fluid coating at low frequency

The same approach can be used to derive low-frequency asymptotic approximations for a thin viscous layer covering the rod. Here, both diverging and converging partial waves from Eq. (3) have to be taken into consideration. The shear drag is then

\[ \tau = i \eta (\nu_1 e^{ikr} - \nu_2 e^{-ikr}). \]  

(9)

The \( \nu_2/\nu_1 \) ratio can be calculated from the boundary conditions at the outside surface of the viscous layer of thickness \( d \), which require that

\[ \tau(d) = i k \eta (\nu_1 e^{ikd} - \nu_2 e^{-ikd}) = 0, \]  

(10)

i.e., \( \nu_2 = \nu_1 e^{ikd} \). At the surface of the rod,

\[
\begin{vmatrix}
(k^2 + s^2) & -2ik & -\frac{\omega \mu l}{\mu_s} (2k^2 + m^2) & -\frac{2\omega \mu l}{\mu_s} k \sqrt{k^2 + m^2} \\
-2ik q & -(k^2 + s^2) & -\frac{2\omega \mu l}{\mu_s} k z & -i \frac{\omega \mu l}{\mu_s} (2k^2 + m^2) \\
q & -ik & k z & ik \\
ik & s & k & \sqrt{k^2 + m^2}
\end{vmatrix} = 0,
\]

(16)

which prompted us to look more closely at the recently published dispersion equation for the leaky Rayleigh mode in Ref. 21. Although it is not explicitly stated in their paper, their solution turns out to be a low-viscosity, low-frequency approximation only since throughout their paper they neglected all but the leading viscosity terms in their expressions. Following Wu and Zhu’s notation, for infinite fluid bath, we derived the full secular determinant as
which should be compared to Eq. (8) in Ref. 21 as \( h \to \infty \). We can conclude that Wu and Zhu’s secular determinant needs certain corrections in the third and fourth elements of the first two rows as well as in the last element of the fourth row to make it rigorous at high frequencies and high viscosities (a separate paper will be published later on the exact solution of the dispersion problem for Rayleigh and Lamb wave propagation in solids loaded by viscous fluids). For the purposes of comparing the dispersion equation of the leaky Rayleigh wave to the asymptotic behavior of the lowest-order longitudinal mode of a cylindrical rod at high frequencies we clearly have to use the full secular determinant of Eq. (16). These corrections are also necessary to obtain accurate results for high-viscosity fluids even at the modest frequencies encountered in numerous applications of practical interest. As an example, Fig. 2 shows the viscosity-induced attenuation of the leaky Rayleigh mode propagating on a SiC substrate immersed in "viscous" water as a function of frequency as calculated from the full solution [Eq. (16)] and the results of Ref. 21. Our analytical model cannot calculate separately the viscosity-induced part of the attenuation; therefore we shall define it as the difference between the respective attenuation coefficients caused by immersion in otherwise identical viscous and viscosity-free fluids. The material parameters used in these and the following calculations are \( c_{11} = 4.258 \times 10^{11} \text{ N/m}^2 \), \( c_{44} = 1.675 \times 10^{11} \text{ N/m}^2 \), and \( \rho = 3.1 \times 10^3 \text{ kg/m}^3 \) for silicon carbide and \( \lambda = 2.25 \times 10^9 \text{ N/m}^2 \) and \( \rho = 10^3 \text{ kg/m}^3 \) for water. In these calculations, in order to demonstrate important but otherwise rather weak effects, we assumed that the viscosity of the fictitious fluid referred to as "viscous" water is \( \eta = 10^{-2} \text{ kg/ms} \), i.e., ten times the normal viscosity of ordinary distilled water (\( \eta = 10^{-3} \text{ kg/ms} \)) at room temperature.

II. NUMERICAL RESULTS

Figure 3 shows the attenuation coefficient of the lowest-order longitudinal mode as a function of frequency for a 150-\( \mu \text{m} \)-diam SiC rod immersed in viscosity-free and "viscous" water. The attenuation coefficient is equal to the imaginary part of the complex axial wave number obtained by numerically solving the dispersion equation. The leaky attenuation is so much higher than the viscous component that the lines essentially overlap each other. At high frequencies, the leaky attenuation coefficient approaches the linear asymptote corresponding to the leaky Rayleigh wave. At about 48 MHz, the dominating leaky attenuation reaches a local maximum. At high frequencies, the attenuation coefficient asymptotically approaches the linear frequency dependence characteristic to the leaky Rayleigh wave propagating along a flat surface. In spite of the increased viscosity of water, the total attenuation of the fluid-loaded fiber is still mainly due to the leaky effect. However, this is not entirely true at low frequencies where the axial surface displacement can be orders of magnitudes stronger than the radial one. Figure 4 shows the same attenuation coefficient previously shown in Fig. 3 at very low frequencies. Up to about 13 MHz, the leaky attenuation in the viscosity-free water is less than half of the total attenuation in "viscous" water, i.e., the viscosity-induced part of the total attenuation of the fluid-loaded rod is higher than the leaky part.

Figure 5 shows the viscosity-induced attenuation of a 150-\( \mu \text{m} \)-diam SiC rod immersed in "viscous" water as a
function of frequency (solid line). Also shown are the low- and high-frequency asymptotic approximations in dotted and dashed lines, respectively. At low and high frequencies, the viscosity-induced attenuation of the lowest-order longitudinal mode closely follows the previously derived asymptotic approximations. At very low frequencies, the attenuation coefficient is proportional to $\omega^{1/2}$ as predicted by Eq. (8). In contrast, at very high frequencies, the attenuation coefficient is approximately proportional to $\omega^{3/2}$.

In the transition region between the expected low- and high-frequency asymptotic behaviors, Fig. 5 reveals a rather unexpected feature; the viscosity-induced attenuation goes through a sharp minimum. At 44.2 MHz, its value drops to 0.04 dB/cm, which is only a fraction of the corresponding values predicted by either asymptotic approximations. The nature and physical origin of this minimum was further investigated by numerical simulations. Figure 6 shows the viscosity-induced attenuation of the 150-μm-diam SiC rod immersed in three waterlike fluids of different viscosities as a function of frequency. The lowest curve corresponds to ordinary water of $\eta = 10^{-3}$ kg/ms, the middle one is the previously used "viscous" water of $\eta = 10^{-2}$ kg/ms, while the highest curve illustrates the case of an even more viscous fluid of $\eta = 10^{-1}$ kg/ms. These results verify that the location of the minimum is not affected by the degree of viscosity of the fluid. The minimum attenuation coefficient was found to be linearly proportional to viscosity, which is a higher power than the ones observed at either low or high frequencies. In this way, the attenuation minimum becomes even more distinct at low viscosities when the fluid interferes less with the vibration of the rod. We can conclude that the observed minimum must be associated with the changing vibration pattern of the lowest-order longitudinal mode in that particular frequency region.

At very low frequencies, the surface vibration of the lowest-order longitudinal mode is essentially linearly polarized in the axial direction, i.e., there is but a negligible normal component with respect to the tangential one. As the frequency increases, the so-called Poisson effect produces more and more radial motion and the surface vibration becomes elliptically polarized with clockwise rotation. At very high frequencies, the same mode asymptotically approaches a Rayleigh-type surface mode that also exhibits elliptical surface vibration polarization but with counter-clockwise rotation. Obviously, there must be a point in the transition region where the surface vibration polarization becomes again purely linearly polarized but this time in the radial direction. At that particular frequency, there is only normal surface vibration as the tangential component completely vanishes. This disappearance of the tangential vibration component has been previously observed, but the phenomenon is not widely known as it bears little importance in most cases except when viscosity-induced losses are considered, which are exclusively caused by the tangential vibration of the surface. The ratio of the tangential and normal surface vibration amplitudes is pure imaginary at all frequencies, which results in the previously mentioned elliptical polarization, and goes through a zero exactly at the location of the previously observed minimum of the viscosity induced attenuation, i.e., at 44.26 MHz.

The sharp minimum in the viscosity-induced attenuation of the lowest-order axial mode was not observed by Liu et al. as their numerical calculations were not extended to high enough frequencies. It should be mentioned that Zhu and Wu recently reported the presence of apparently similar sharp minima in the viscosity-induced attenuation spectra of fluid-coated plates, but those minima are associated with resonance vibrations of the thin fluid layer and the resonance frequencies are determined by the thickness and material properties of the fluid rather than by those of the solid. However, since the surface vibration polarization of the lowest-order symmetric Lamb mode of a solid plate exhibits a similar behavior to that of the lowest-order longitudinal mode in a solid rod, it is expected that the viscosity-induced attenuation of the Lamb mode in a thin plate immersed in infinite viscous fluid also exhibits a sharp minimum as the frequency is increased.

As the observed minimum in the viscosity-induced part of the total attenuation is clearly caused by the disappearance of the tangential component of the surface vibration at that
particular frequency, essentially the same effect is expected to occur when the rod is covered by a thin layer of viscous fluid. Figure 7 shows the comparison between the exact solution and the asymptotic approximation [Eq. (15)] for a 2.35-mm-diam steel rod covered with a 100-μm-thick layer of highly viscous fluid. The material parameters used in these calculations were $c_{11} = 2.5 \times 10^{11}$ N/m², $c_{44} = 8 \times 10^{11}$ N/m², and $\rho = 7.9 \times 10^3$ kg/m³ for steel and $\eta = 40$ kg/ms, $\lambda = 2.25 \times 10^9$ N/m², and $\rho_l = 10^3$ kg/m³ for the high-viscosity fluid (these parameters were chosen to approximate an experimental case to be described later). The attenuation minimum at 1.2 MHz is clearly visible. It is interesting that the low-frequency asymptotic behavior of the viscosity-induced attenuation of the fluid-coated rod is markedly different from that of the immersed one. For a rod immersed in an infinite fluid bath, the attenuation coefficient was proportional to the square roots of frequency and viscosity, as shown by Eq. (8). In comparison, for a rod covered by a thin fluid layer, the attenuation coefficient is proportional to the square of frequency and inversely proportional to viscosity, as shown by Eq. (15). It is somewhat surprising that, in contrast with immersion in an infinite fluid bath, for thin coating the viscosity-induced attenuation decreases with increasing viscosity. The reason for this is that, in the case of a thin coating, increasing viscosity assures that the whole fluid layer moves in union with the fiber. In this way, the fluid coating represents a significant mass loading but essentially no friction losses occur.

III. EXPERIMENTAL RESULTS

The primary goal of our experimental investigation was to verify our main analytical results for the viscosity-induced attenuation of the lowest-order longitudinal mode in a thin rod. The experimental arrangement is described in detail in Ref. 2. In large diameter rods ($d > 250$ μm), end-on excitation and detection by contact longitudinal transducers can be applied quite easily. The wire or fiber can be mounted on the face plate of the transducer by a small drop of quick-setting adhesive. Smaller diameter specimens ($d < 250$ μm) are rather difficult to mount in this way, therefore contact shear transducers were used. The specimen is laid flat on the face plate of the transducer parallel to the polarization direction and fixed by a small drop of instant glue. First, the transmitted signal through the free rod is recorded and its spectrum is calculated by FFT. When we submerge the specimen, the measured attenuation is always a little higher than the one measured when the specimen is taken out of the fluid. This is because, in the second case, the surface of the specimen is still wet, i.e., coated with a fluid layer. This layer is much thinner than the acoustic wavelength but still thicker than the viscous skin depth at ultrasonic frequencies. Of course, the fluid quickly evaporates from the surface in a few minutes and the viscous loss also disappears. When the rod is submerged, we can compare the transmitted signals through the dry and immersed specimens to determine the total attenuation caused by fluid loading. Alternatively, when the rod is taken out from the fluid bath, we can compare the transmitted signals through the immersed and fluid-coated specimens to determine the leaky attenuation only. Finally, the viscosity-induced part can be determined by comparing the transmitted signals measured before and after immersion, i.e., on the dry and fluid-coated specimens.

First we tried to measure the viscosity-induced attenuation of the lowest-order longitudinal mode in an SCS-6 fiber. This fiber is one of the most frequently used reinforcements in state-of-the-art composite materials. It is composed of a carbon core, a relatively thick SiC cladding, and a very thin carbon-rich coating layer, and its outside diameter is approximately 150 μm. Figure 8 shows the measured and calculated viscosity-induced attenuations of the lowest-order longitudinal-mode in an 80-cm-long SCS-3 fiber up to 10 MHz. At such low frequencies, the fiber could be modeled as a homogeneous SiC fiber without any significant effect on the viscosity-induced attenuation. The theoretical curve was obtained by matching the experimental data via varying the viscosity of the fluid. The best fitting parameter was found to be $\eta = 1.23 \times 10^{-3}$ kg/ms, slightly higher than the tabulated viscosity of water at room temperature. This discrepancy could be caused by the Photo-Flo 200 solution (made by the Eastman Kodak Company) which we had to add to the water to
assure that the fluid wets the surface of the slightly hydrophobic fiber. As expected, the measured attenuation is proportional to the square root of frequency over most of the frequency range. At high frequencies, there is a small excess attenuation probably caused by the appearance of the emerging leaky loss as the thickness of the fluid layer becomes significant with respect to the acoustic wavelength in the fluid.

Our efforts directed at observing the attenuation minimum predicted at around 44 MHz in such fibers proved to be futile because of the substantial intrinsic attenuation of the fiber at high frequencies. Shorter and shorter fibers were selected so that the inspection frequency could be increased, but even free fibers as short as 1–2 in. presented attenuations in excess of 30–50 dB around 40 MHz, which effectively eliminated the possibility of observing this interesting phenomenon on a real fiber. Since the intrinsic attenuation of the fiber increases much faster than linearly with frequency, the phenomenon could be more easily observed in larger-diameter rods at low frequencies. Therefore, we decided to use a 2.35-mm-diam steel wire that exhibits the same phenomenon slightly above 1 MHz (see Fig. 7). Naturally, in order to assure a significant viscous loss at such low frequencies, we had to use a highly viscous fluid such as honey. First, we measured the group velocity of the wave at different discrete frequencies by using the tone-burst excitation method described in Ref. 2. Figure 9 shows the measured and calculated group velocity of the lowest-order longitudinal mode in the 80-cm-long free wire. The actual elastic stiffnesses of the steel wire were determined from this experimental data by best fitting the theoretical curve to the measured data. These parameters, which were previously listed and used in connection with Fig. 7, were then used in our estimation of the viscous losses as the fiber was covered with a thin layer of commercial honey.

Figure 10 shows the measured and calculated viscosity-induced attenuations of the lowest-order longitudinal mode in the 2.35-mm-diam honey-coated steel wire. The viscosity of honey was estimated at $\eta = 35$ kg/ms and the somewhat uncertain thickness of the rather uneven honey coating was obtained by best fitting of the theoretical curve to the experimental data. This matching yielded $d = 80$ $\mu$m, a fairly reasonable value considering the amount of honey used to cover the wire. Although the very close quantitative agreement might be due to the adjustments made in the uncertain parameters of the honey coating in order to achieve the best fitting, the experimental results are clearly consistent with the main features of the theoretically predicted behavior. Most importantly, the measured data clearly verifies the presence of the sharp minimum in the viscosity-induced attenuation at the predicted frequency. This feature is not only conceptually interesting, but also might be very important from a practical point of view. Embedded fibers and wires can be used to monitor epoxy curing during composite manufacturing. However, the measured attenuation depends on both the compressibility and viscosity of the surrounding medium to be monitored and separation of these parameters is very difficult. The presence of the sharp minimum in the viscosity-induced component can be exploited to separate these additive effects. Monitoring the attenuation of the lowest-order longitudinal mode at this specific frequency of the embedded waveguide allows us to assess the matrix compressibility independent of the simultaneously increasing viscosity (or shear modulus) of the epoxy. The viscosity parameter then can be assessed from a second measurement made at a different, preferable lower frequency.

IV. CONCLUSIONS

Our previously introduced unified analytical treatment of longitudinal guided wave propagation along the fiber direction of multilayered coaxial fibers has been extended to viscous fluids. Newtonian fluids were simply modeled as hypothetical isotropic solids having rigidity $c_{44} = -i \omega \eta$, where $\eta$ denotes the viscosity of the fluid and $\omega$ is the angular frequency. In this way, the vorticity mode associated with the viscosity of the fluid is formally described as the shear mode in the hypothetical solid. Either free, immersed, or embedded fibers can be studied by this technique. In the specific cases considered in this paper, the fiber was either immersed in an infinite bath or coated with a thin layer of

FIG. 9. Measured and calculated group velocity of the lowest-order longitudinal mode in a 2.35-mm-diam free steel wire.

FIG. 10. Measured and calculated viscosity-induced attenuations of the lowest-order longitudinal mode in a 2.35-mm-diam honey-coated steel wire ($d = 80$ $\mu$m, $\eta = 35$ kg/ms).
viscous fluid. Low-frequency asymptotic approximations were derived to gain better insight into the viscous attenuation mechanism. For a thin rod immersed in an infinite fluid bath, the attenuation coefficient was found to be proportional to the square roots of frequency and viscosity. In comparison, for a similar rod covered by a thin fluid layer, the attenuation coefficient is proportional to the square of frequency and inversely proportional to viscosity. It was also shown that the analytical results previously available in the literature for leaky Rayleigh waves must be modified in order to obtain accurate results for high values of viscosity and frequency. Our analytical predictions for the viscosity-induced attenuation of the lowest-order longitudinal mode in a rod were shown to be in good agreement with the derived asymptotic approximations at low and high frequencies. Generally, our experimental results on immersed and fluid-covered fibers and wires showed good agreement with the analytical predictions. In the transition region, the exact theory and the experiments both show a sharp minimum in the viscosity-induced attenuation of the lowest-order longitudinal mode. This minimum occurs at a particular frequency when the otherwise elliptical polarization of the surface vibration becomes linearly polarized in the radial direction. The presence of this minimum can be exploited to separate the leaky and viscosity-induced contributions in the total attenuation of the lowest-order longitudinal mode propagating in a fluid-loaded rod.


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