Focal shift of convergent ultrasonic beams reflected from a liquid–solid interface

Peter B. Nagy, a) Kangra Cho, and Laszlo Adler
Department of Welding Engineering, The Ohio State University, Columbus, Ohio 43210
D. E. Chimenti
Materials Laboratory, Air Force Wright Aeronautical Laboratories, Wright-Patterson Air Force Base, Dayton, Ohio 45433

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It is shown by schlieren photography that a convergent acoustic beam reflected from a liquid–solid interface can exhibit a focal shift in the axial direction when the angle of incidence deviates slightly from the critical angle. The results confirm the recent predictions of Bertoni et al. [Trait. Sign. 2, 201–205 (1985)] about this interesting detail of the general Rayleigh angle phenomenon.

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INTRODUCTION

In 1947, Goos and Hanchen reported the first experimental evidence of the lateral displacement of a light beam upon total internal reflection. Only a few years later, Schoch found the acoustical equivalent of this phenomenon by schlieren photography. Schoch’s results showed that the center of an ultrasonic beam reflected from a liquid–solid interface is displaced by a certain amount along the interface if the angle of incidence is close to the Rayleigh angle. This nonspecular reflection can be easily explained by the angular spectrum representation of the acoustic beam and the angular dependence of the reflection coefficient of the interface. The so-called Schoch displacement was found to be proportional to the first derivative of the phase of the reflectance. Later, McGuirk and Carniglia showed that the same angular spectrum representation approach predicts the shift of the reflected optical beam along its direction of propagation, too, and this displacement is proportional to the second derivative of the phase of the reflectance. It should be mentioned that a simple ray model can yield basically the same result for the axial shift, too. The acoustic field changes much slower along the axis of the beam than in the lateral direction; therefore, the axial shift is usually negligible in spite of its absolute value being much higher than that of the lateral displacement. In a recent article, Bertoni et al. suggested that the acoustical equivalent of this axial shift might be of some importance in the case of convergent, i.e., focused, ultrasonic beams. In the following, we shall give a short analysis of their results to find the optimal conditions for experimental detection of the predicted axial shift and show the effect by schlieren photography.

I. NONSPECULAR REFLECTION OF CONVERGENT BEAMS

Let us have an acoustic beam impinging on a liquid–solid interface at an angle of incidence \( \theta_i \). The incident and reflected fields are expressed in the \( x,z \) and \( x',z' \) coordinate systems, respectively (see Fig. 1). Let the acoustic field in the \( z = 0 \) plane be given by \( p(x,0) \); then the angular spectrum in this plane can be written as follows:

\[
A_0(k_x) = \frac{1}{2\pi} \int p(x,0) e^{-i k_x x} dx.
\]

(1)

Only the phase of the angular spectrum will change as the beam propagates in a homogeneous, lossless medium

\[
A(k_x,z) = A_0(k_x) e^{i k} z',
\]

(2)

where \( k = \sqrt{k_x^2 - k_z^2} \) and \( k \) denotes the wavenumber in the fluid. Let us assume that there exists a well-defined amplitude reflectance \( r(k_x) \) of the fluid–solid interface, so the angular spectrum of the reflected field in the \( z' = 0 \) plane can be written as

\[
A_0'(k_x') = r(k_x) A_0(k_x) e^{i k} z',
\]

(3)

where \( z_0 \) is the distance from the \( z = 0 \) plane to the interface and then to \( z' = 0 \). Furthermore, let us assume that the angle of incidence is sufficiently higher than the shear critical angle, so for all spatial frequency components of the incident beam the reflection coefficient is unit in amplitude

\[
r(k_x) = e^{i \psi(k_x)},
\]

(4)

where \( \psi(k_x) \) is the phase of the reflectance. Expanding \( \psi(k_x) \) in a Taylor series about \( k_x = 0 \), we can approximate the phase by the first three terms:

\[
\psi(k_x) = \psi_0 - k_x S + k_x^2 (F/2k) + \cdots,
\]

(5)

where

\[
\psi_0 = \psi(0),
\]

(6)

\[
S = \left( \frac{- \partial \psi(k_x)}{2k_x} \right)_{k_x = 0},
\]

(7)

and

\[
F = \left( k \frac{\partial^2 \psi(k_x)}{\partial k_x^2} \right)_{k_x = 0}.
\]

(8)

a) Also at applied Biophysics Laboratory, Technical University, Budapest, Hungary.
FIG. 1. Coordinate systems.

The third term in Eq. (5) can be further simplified for \( k_x \ll k \), when
\[
k_x = \left( k^2 - k_x^2 \right)^{1/2} \approx k - \left( k_x^2 / 2k \right),
\]
so
\[
\psi(k_x) = \left( \psi_0 + kF \right) - k_x S - k_z F.
\]
Substituting Eq. (10) into Eq. (3), and inverse Fourier transforming \( A(k_x) \) yields
\[
p'(x',0) = e^{i(\theta + k_x l)} \int \exp \left\{ i \left[ (x' - S)k_x + (z' - F)k_z \right] \right\} dk_x.
\]
It is easy to recognize from Eq. (11) that, apart from the less important phase term, the reflected field is simply the specular reflection shifted along the \( x' \) and \( z' \) axes by \( S \) and \( F \), respectively (see Fig. 2):
\[
p'(x',z') = e^{i(\phi + kF)} p(x' - S, z' - F). \tag{12}
\]
This is, of course, a very rough approximation of the actual reflected field that results from the redistribution of energy due to the double mode conversion occurring around the Rayleigh angle. Generally, the beam profile becomes highly distorted, or even split. Our immediate aim is to determine the special conditions under which the redistribution can be expressed in terms of lateral and axial displacements; therefore, we shall analyze the phase of the reflection coefficient in more detail.

II. DISPLACEMENT APPROXIMATION

When calculating the lateral and axial displacements, some difficulty arises from the choice of our coordinate system. The reflection coefficient is much easier to express in a coordinate system fixed to the interface plane, e.g., by the projection \( k_x \) of the wave vector \( k \) in this plane:
\[
k_x = k_x \cos \theta_1 + \left( k^2 - k_x^2 \right)^{1/2} \sin \theta_1. \tag{13}
\]
For \( k_x \ll k \), Eq. (13) can be approximated by
\[
k_x = k_x \cos \theta_1 + k \sin \theta_1. \tag{14}
\]
We shall use the Bertoni–Tamir\(^7\) approximation for the reflection coefficient:
\[
r(k_x) = (k_x - k_0)/(k_x - k_p), \tag{15}
\]
where, without dissipation in the solid, \( k_0 \) and \( k_p \) are complex conjugates
\[
k_0 = k_R - i\alpha, \tag{16}
\]
and
\[
k_p = k_R + i\alpha. \tag{17}
\]
Here, \( k_R \) is the wavenumber of the Rayleigh wave on the free solid surface and \( \alpha \) is representing the energy loss of the surface wave due to leaking into the liquid. Figure 3 shows the excellent approximation given by Eq. (15) for the phase of the reflection coefficient. In a somewhat indirect, but more practical, way, we compared the Schoch displacement calculated by Eq. (15) to the exact solution for aluminum at 10 MHz. On the other hand, we must emphasize that the introduction of this approximation is more of a question of convenience than necessity, and we choose it mainly to follow the original derivation of Bertoni et al.\(^5\) The phase of the reflectance can be written from Eq. (15) as follows:
\[ \psi(k_z) = 2 \arctan \left[ \frac{\alpha_i}{(k_z - k_R)} \right], \]  
which yields, by using Eqs. (7) and (14),

\[ S = \left\{ 2 \alpha_i \left[ (\sin \theta_i - \sin \theta_R)^2 + k^2 \right] \right\} \cos \theta_i, \]  
where we introduced the Rayleigh angle \( k_R = k \sin \theta_R \).

Equation (19) can be written into a more familiar form by \( \Delta S = 2/\alpha_i \), the maximum lateral displacement along the interface occurring at the Rayleigh angle,

\[ \Delta S = \frac{\Delta S \cos \theta_i}{1 + \left[ \left( \frac{\Delta S k}{2} \right)^2 (\sin \theta_i - \sin \theta_R)^2 \right]}. \]  

Figure 3 shows the characteristic bell-shaped curve of \( S(\theta_i) \). The same curve indicates the inherent limitations of describing the energy redistribution by the Schoch displacement. The actual reflected field cannot be produced by simply displacing the specular reflection unless all spatial frequency components contributing to it are displaced by the same amount; i.e., there is no dispersion. In other words, the bandwidth of the angular spectrum must be less than that of the Schoch displacement versus angle of incidence curve, which means that the diameter of an unfocused beam must be larger than \( \Delta S \), which is in agreement with the conclusion of Bertoni and Tamir’s detailed analysis.

We shall use a similar approach to study the feasibility of the above-introduced axial displacement approximation to describe the redistributed field. From Eqs. (8), (14), and (20), we can write that

\[ F = k \cos^2 \theta_i \frac{\Delta S^3/2 k (\sin \theta_i - \sin \theta_R)}{1 + \left[ \left( \frac{\Delta S k}{2} \right)^2 (\sin \theta_i - \sin \theta_R)^2 \right]^2}. \]  

Figure 4 shows the axial shift versus angle of incidence curve for aluminum at 10 MHz. The axial shift is negative below the Rayleigh angle and positive above it; furthermore, there is no axial shift at the Rayleigh angle, where the lateral one achieves its maximum. When evaluating this curve, we must remember our former argument about the feasibility of this rough approximation; i.e., the actual reflected field cannot be described in terms of simple displacements unless all spatial frequency components contributing to it are shifted by about the same amount. Unfortunately, the axial displacement changes very sharply with the angle of incidence, which indicates a more complex distortion of the beam profile rather than a simple displacement. The sharp peaks of \( F(\theta_i) \) are only a small \( \Delta \theta_p \) angle below and above the Rayleigh angle, where, from the first derivative of Eq. (21),

\[ \Delta \theta_p = 2/3 \Delta S k. \]  

Table I shows the Rayleigh angle \( \theta_R \) and peak separation \( \Delta \theta_p \) parameters at a water–solid interface for different solid materials of practical importance. Just from the extremely small separation between the peaks, it is quite clear that the focal shift of the convergent ultrasonic beam will be very difficult to detect. The focused beam should be so well collimated, with an angle of convergence of only about 0.2°–0.3°, that the position of the focal point will be more blurred than

<table>
<thead>
<tr>
<th>Material</th>
<th>Rayleigh angle ( \theta_R )</th>
<th>Peak separation ( \Delta \theta_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>30.7°</td>
<td>0.49°</td>
</tr>
<tr>
<td>Aluminum oxide</td>
<td>14.7°</td>
<td>0.09°</td>
</tr>
<tr>
<td>Brass</td>
<td>48.0°</td>
<td>0.57°</td>
</tr>
<tr>
<td>Copper</td>
<td>42.0°</td>
<td>0.38°</td>
</tr>
<tr>
<td>Glass</td>
<td>31.0°</td>
<td>0.45°</td>
</tr>
<tr>
<td>Molybdenum</td>
<td>28.3°</td>
<td>0.11°</td>
</tr>
<tr>
<td>Stainless steel</td>
<td>30.7°</td>
<td>0.18°</td>
</tr>
<tr>
<td>Titanium</td>
<td>30.5°</td>
<td>0.31°</td>
</tr>
</tbody>
</table>

\[ \text{FIG. 4. Calculated axial displacement versus angle of incidence for aluminum at 10 MHz.} \]

\[ \text{FIG. 5. Calculated beam radius versus axial distance for a convergent Gaussian beam of 100-mm geometrical focal length and 10-mm diameter.} \]
the possible axial shift. As a matter of fact, an unfocused beam of 10 mm in diameter is not so well collimated up to about 40 MHz due to diffraction (finite beam) effects, let alone a focused one. In other words, to collimate the acoustic beam to such a degree, we should increase the frequency so high that the focal shift, which is inversely proportional to frequency, would be negligible.

III. EXPERIMENTAL RESULTS

It seems to be a more efficient strategy to move the angle of incidence farther from the Rayleigh angle, where the axial displacement, although being somewhat smaller, changes much slower with angle. In this case, we can use less collimated beams of sharper focusing at relatively low frequencies. To illustrate this, we showed the beam radius versus axial distance curves of a Gaussian radiator at different frequencies in Fig. 5. The 10-mm-diam transducer has a geometrical focal length of 100 mm. At 10 MHz, the focal point seems to be sharp enough; the beam diameter increases from about 2 mm to about 4 mm within ± 40 mm. To check the feasibility of the displacement approximation, we calculated the redistributed field by a computer for Gaussian beam distribution. The results (see Fig. 6) confirm that the axial shift is readily detectable in this case, although some distortion of the beam profile is present. Furthermore, the actual focal shift seems to be in very good quantitative agreement with the simple displacement approximation, as is shown in Fig. 3.

Figure 7 shows the schlieren photographs of the incident and reflected beams at five different angles of incidence. According to Fig. 4, the calculated focal shift is negligible at 23.5°, 30.5°, and 37.5°, and about ± 25 mm at 28.5° and 32.5°, respectively. The numerical predictions, of course, cannot be taken too seriously, because the relatively high convergence of the focused beam causes strong aberrations. On the other hand, the focal shift is quite evident from the pictures at 28.5° and 32.5°. What is more, its value seems to be a little higher than the predicted 25 mm (the optical diameter is 60 mm), which is probably due to the above-mentioned aberrations. The widening of the reflected beam at the Rayleigh angle shows the insufficiency of the displacement approximation, although the well-known beam splitting is not present yet.
Unfortunately, this widening further reduces the visibility of the otherwise barely detectable waist region, so the predicted lack of focal shift at the Rayleigh angle is less clearly demonstrated. The schlieren pictures were taken of a cylindrically focused 10-mm-diam transducer of 100-mm geometrical focal length at 10 MHz from a smooth aluminum surface.

IV. CONCLUSIONS

To our knowledge, this is the first report to verify, by experimental means, the existence of focal shifts in the reflected field of convergent ultrasonic beams at a fluid–solid interface. In the vicinity of the Rayleigh angle, the otherwise specular reflection becomes greatly distorted due to the strong excitation of a leaky surface wave. The interference between the specularly reflected part and the double-mode converted reradiated part usually results in an entirely new beam profile. Under certain conditions, the distortion is small enough to describe simply in terms of lateral and axial displacements. This is the case when a relatively wide, well-collimated beam of long focal length is reflected from a solid of small Schoch displacement. Schlieren photographs of the reflected beam from a water–aluminum interface at 10 MHz show good qualitative agreement with Bertoni et al.’s predictions. Although being a relatively weak effect, the focal displacement of convergent ultrasonic beams upon reflection from a liquid–solid interface is an interesting aspect of the so-called Rayleigh angle phenomenon caused by the strong excitation and reradiation of the leaky Rayleigh waves.

ACKNOWLEDGMENT

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2A. Schoch, Arch. Exakten Naturwiss. 23, 127–234 (1950); also Acustica 2, 18–19 (1952).