Slow wave propagation in air-filled permeable solids

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The propagation of slow compressional waves in air-saturated permeable solids was studied by experimental means between 10 and 500 kHz. The velocity and attenuation coefficient were measured as functions of frequency from the insertion delay and loss of airborne ultrasonic waves transmitted through thin slabs of 1–5 mm in thickness. Porous ceramics of 2–70 Darcy and natural rocks of 200–700 mDarcy permeability were tested. In the low-frequency (diffuse) regime, the experimental results are consistent with theoretical predictions; the phase velocity and attenuation coefficient are essentially determined by the permeability of the specimen and both increase proportionally to the square root of frequency. In the high-frequency (propagating) regime, the experimental results are consistent with the theoretical predictions for the phase velocity but not for the attenuation coefficient. The phase velocity asymptotically approaches a maximum value determined by the tortuosity of the specimen while the attenuation coefficient becomes linearly proportional to frequency instead of the expected square-root relationship. It is suggested that the observed discrepancy is due to the irregular pore geometry that significantly reduces the high-frequency dynamic permeability of the specimens.

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INTRODUCTION

The most interesting feature of acoustic wave propagation in fluid-saturated porous media is the appearance of a second compressional wave, the so-called slow wave. The existence of a slow compressional wave in an isotropic and macroscopically homogeneous fluid-saturated porous medium was predicted by Biot in 1956, 1,2 The main characteristic of this mode is that its velocity is always lower than both the compressional wave velocity in the fluid and the longitudinal velocity in the solid frame. Below a critical frequency, which depends on the pore size in the frame and the viscosity of the fluid, the slow compressional wave is highly dispersive and strongly attenuated over a single wavelength. Above this critical frequency, it becomes a dispersion-free propagating wave with increasing but fairly low attenuation. The slow compressional wave represents a relative motion between the fluid and the solid frame. This motion is very sensitive to the viscosity of the fluid and the dynamic permeability of the porous formation. Naturally, low-viscosity liquids such as water are the fluids most often used in experimental studies of slow wave propagation. However, it will be shown in this paper that it also makes good sense to use gaseous fluids such as air to saturate the porous specimens.

Since 1980, when Plona was able to observe slow wave propagation in water-saturated porous ceramics, 3 the question of why slow waves cannot be detected in real rocks has been one of the major issues in the acoustics of fluid-saturated materials. Klimenatos and McCann showed that this lack of perceivable slow wave propagation is probably due to inherent internal impurities, such as submicron clay particles, found in all types of natural rocks. 4 These clay particles are deposited within the pore throats and on the surfaces of the rock grains as well as suspended in the fluid. The clay content greatly increases viscous drag between the fluid and solid frame, which results in excessive attenuation and usually complete disappearance of the slow wave. Recently, Gist suggested a simple model to demonstrate that, due to inherent pore-wall roughness, the attenuation of the slow compressional wave in rocks can be many times higher than the predictions of the Biot theory. 5 The excess viscous drag explains the difficulty to detect the slow wave in fluid-saturated rocks as well as the observed correlation between ultrasonic attenuation and clay content in sandstones. One way to reduce the excessive attenuation of slow waves in porous materials is to use special fluids of very low viscosity to saturate the specimen. For instance, superfluid 4He below 1.1 K has been shown to work very well in fused glass bead samples, 6 superleak materials consisting of compacted powders, 7–9 and in sandstones. 10

The question of whether or not excessive attenuation in viscous fluid-saturated natural rocks renders the detection of slow waves impossible arises. Not necessarily! Even a very weak slow wave, attenuated by as much as 50–60 dB, could easily be detected but for the presence of much stronger background “noise” caused by the direct arrivals and scattered components of the fast compressional and/or shear waves. If we could generate the slow wave only and nothing else, it would be much easier to detect in spite of the substantial attenuation. Compared to the solid frame, liquids like water usually have a lower, but still comparable density \( \rho_f \) and bulk modulus \( B_f \). Although their viscosity \( \mu \) is also relatively high, which makes saturation of the porous sample somewhat troublesome, their kinematic viscosity \( \eta = \mu / \rho_f \) is fairly low. On the other hand, gaseous fluids like air have very low density, bulk modulus, and viscosity, while their kinematic viscosity is usually rather
high. Therefore, it is very simple to saturate a porous sample by air, but the slow wave is expected to be highly dispersive and strongly attenuated. In spite of these adverse effects, slow waves can be readily observed in air-filled porous samples, including certain natural rocks, by using airborne ultrasonic waves. In the case of air saturation, because of the tremendous acoustical mismatch between the incident compressional wave and the porous solid, all the incident energy is either reflected or transmitted via the slow wave without generating appreciable fast compressional or shear transmitted waves. In order to demonstrate this crucial effect, Fig. 1 shows the slow, fast, and shear wave transmission coefficients through water- and air-saturated glass bead specimens. The physical parameters of the glass bead specimen and details of the calculation are given in Ref. 12.

In the case of air saturation, because of the tremendous acoustical mismatch between the incident compressional wave and the porous solid, all the incident energy is either reflected or transmitted via the slow wave without generating appreciable fast compressional or shear transmitted waves. In order to demonstrate this crucial effect, Fig. 1 shows the slow, fast, and shear wave transmission coefficients through water- and air-saturated glass bead specimens. The physical parameters of the glass bead specimen and details of the calculation are given in Ref. 12. In the case of water saturation, the slow compressional wave is usually 5–10 dB weaker than the fast compressional or shear modes and it is much more attenuated. Also, because of its lower velocity, it arrives later than the other modes and it is often overshadowed by multiple reflections and scattered components of these stronger signals. Maybe the only exception is when the shear velocity is sufficiently high so that we can work above the second critical angle where the slow compressional wave becomes the only propagating mode in the fluid-saturated sample (unfortu-

nately, this does not happen in most natural rocks where the shear velocity is rather low). On the other hand, in the case of air saturation, the slow compressional wave is at least 70 dB stronger than all other modes and, due to the very low sound velocity in air, the shear critical angle drops below 15°, above which only the slow wave is transmitted through the sample. This means that the highly attenuated slow wave is submerged in electrical noise rather than spurious signals so it can be easily recovered by simple time-averaging.

In spite of the excellent coupling between the incident compressional wave and the transmitted slow wave and the obvious advantage of saturating the specimen with low-viscosity air rather than high-viscosity water, slow wave propagation in air-filled porous samples has never been extensively studied above the critical frequency where it becomes a propagating mode. It should be mentioned that considerable work has been done on air-filled porous materials in the so-called diffuse regime, i.e., at relatively low frequencies between 50 Hz and 4 kHz. Above the critical frequency, slow waves are not expected to propagate in air-saturated porous samples as well as in water-saturated ones. Since the kinematic viscosity of air is so large and the velocity of sound so small, there is but a very narrow frequency window where the attenuation coefficient is sufficiently low to observe a more or less dispersion-free, scattering-free slow wave. This “window” is set by the conditions that the viscous skin depth \( \delta = (2\eta/\omega)^{1/2} \), where \( \omega \) denotes the angular frequency, be less than the pore size \( a_p \) and, simultaneously, the wavelength \( \lambda \) be larger than the grain size \( b_g \).

In Table I, the limits of the frequency window where slow wave propagation is expected. \( a_p = 200-\mu\text{m} \) grain diameter and \( \phi = 30\% \) porosity were assumed in the calculations. By assuming a perfectly stiff frame, the high-frequency asymptotic value of the slow wave velocity can be easily calculated as \( v = v_f/\tau^{1/2} \), where \( v_f \) denotes the sound velocity in the fluid and \( \tau \) is the tortuosity of the permeable frame. For the purposes of the example given in Table I, the tortuosity was estimated from the porosity as \( \tau = 1/2(\phi^{-1} + 1) \approx 2.17 \). To determine \( f_{min} \) and \( f_{max} \), we assumed that the pore size is approximately six times smaller than the grain size \( a_p \approx a_g/6 \) and at least four times larger than the viscous skin depth \( a_p > 4\delta \) to account for the smaller cross sections at the crucial pore throats:

\[
f_{min} = 576\eta/\pi a_g^2
\]
\[ f_{\text{max}} = \frac{v_f}{2\pi a_g} t^{1/2}. \]  

Table I clearly demonstrates the greatly reduced frequency window where more or less dispersion-free and attenuation-free slow wave propagation can be expected in typical air-filled samples of approximately 200-μm grain size. On the other hand, these results do not exclude slow wave propagation over a much larger frequency range. They simply mean that the slow wave becomes increasingly dispersive below 100 kHz and very strong attenuation can be expected above 200 kHz.

### I. THEORETICAL BACKGROUND

For the special case of air-saturated permeable solids of random formation, Attenborough's theoretical model can be used to determine both the complex wave number \( k \) and the complex acoustic impedance \( Z \):

\[ k(\omega) = \omega \left( \frac{\rho(\omega)}{K(\omega)} \right)^{1/2} \]  

\[ Z(\omega) = \left( \frac{\rho(\omega) K(\omega)}{2} \right)^{1/2}, \]  

where \( \rho(\omega) \) and \( K(\omega) \) denote the complex density and the complex modulus, respectively, of the air-saturated material. The complex density includes the effect of viscosity

\[ \rho(\omega) = \frac{\rho_{\text{ref}}}{\phi[1 - \tau(\xi)]}, \]  

while the complex modulus includes the somewhat weaker effect of heat conduction in air,

\[ K(\omega) = \frac{K_f}{\phi[1 + (\gamma - 1)\tau(\Pr^{1/2}\xi)]}. \]  

Here, \( \gamma \) denotes the specific heat ratio, \( \Pr \) is the Prandtl number \((\gamma \approx 1.4 \text{ and } \Pr \approx 0.74 \text{ for air}) \), and \( \tau \) is a simple function of the normalized pore radius \( \xi \):

\[ \tau(\xi) = \frac{2J_1(\sqrt{-i\xi})}{\sqrt{-i\xi} J_0(\sqrt{-i\xi})}, \]  

where \( J_0 \) and \( J_1 \) are the zero- and first-order Bessel functions and \( i \) is the imaginary unit. For cylindrical tubes, the normalized pore radius is exactly known:

\[ \xi = a(\omega/\eta)^{1/2}, \]  

where \( a \) denotes the actual pore radius. For real porous materials of random pore geometry, there is no corresponding exact solution. Attenborough suggested that the normalized pore radius should be calculated as

\[ \xi = (2\tau_\omega \kappa_0 \phi / \gamma s_p)^{1/2}, \]  

where \( \kappa_0 \) is the static permeability, \( \tau_\omega \) is the real-valued high-frequency tortuosity, and \( s_p \) is the so-called pore shape factor ratio that is usually between 0.1 and 0.5.

Attenborough's analytical technique provides a unified model for both low-frequency (diffuse) and high-frequency (propagating) regimes of the slow compressional wave in air-saturated porous solids. It uses four basic parameters, namely porosity \( \phi \), high-frequency tortuosity \( \tau_\omega \), static permeability \( \kappa_0 \), and pore shape factor ratio \( s_p \), to describe the porous formation. This model works better for air-saturated materials than the dynamic permeability model of Johnson et al. because the latter one is limited to fluids of negligible thermal expansion coefficient. Comparison of the two models reveals that the actual difference between them is rather small. The best agreement can be achieved by asymptotic matching at low and high frequencies, which requires that \( s_p = \gamma^{1/2} \approx 0.423 \) and \( \Lambda \approx (5.6\tau_\omega \kappa_0 / \phi)^{1/2} \), where \( \Lambda \) is a measure of the pore size in the notation of Ref. 20. As an example, Fig. 2 shows the calculated attenuation coefficient of the slow compressional wave in a porous solid of \( \phi = 0.3, \tau_\omega = 1.79, \) and \( \kappa_0 = 2.2 \times 10^{-12} \text{ m}^2 \). The largest discrepancy between the two models occurs at the transition between the diffuse and propagating regimes, but it never exceeds 12%.

Since the static permeability \( \kappa_0 \) and the pore shape factor ratio \( s_p \) always occur in the same combination through the normalized pore radius \( \xi \), they cannot be separated by acoustical measurements. According to the dynamic permeability model, at low frequencies, the complex wave number of the slow compressional wave can be approximated as

\[ k_{\text{low}} \approx k_f (i\phi \eta / \kappa_0 \omega)^{1/2}, \]  

Atttenborough's model yields a similar form of

\[ k_{\text{low}} \approx k_f (i\phi \eta / \kappa_0 \omega)^{1/2}, \]  

where \( \kappa_a \) denotes the "acoustic" permeability:

\[ \kappa_a = \kappa_0 / (4\gamma s_p^2). \]  

According to Attenborough's model, only three independent parameters can be determined from acoustic measurements on air-filled porous solids: porosity \( \phi \), high-frequency tortuosity \( \tau_\omega \), and the low-frequency acoustic permeability \( \kappa_a \), which is a combination of the static permeability \( \kappa_0 \) and the pore shape factor ratio \( s_p \). The latter one is always less than 0.5 while \( \gamma \) is higher than one for

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FIG. 2. Comparison between the slow wave propagation models of Attenborough (Ref. 16) and Johnson et al. (Ref. 20).
TABLE II. Estimated material parameters of cemented glass bead specimens (φ=30% and τ0=1.79 for all grades).

<table>
<thead>
<tr>
<th>Grade</th>
<th>( K_0/(10^{-12} \text{m}^2) )</th>
<th>( s_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>2.2</td>
<td>0.460</td>
</tr>
<tr>
<td>40</td>
<td>6.5</td>
<td>0.460</td>
</tr>
<tr>
<td>55</td>
<td>11</td>
<td>0.475</td>
</tr>
<tr>
<td>90</td>
<td>27</td>
<td>0.475</td>
</tr>
<tr>
<td>175</td>
<td>67</td>
<td>0.475</td>
</tr>
</tbody>
</table>

gases. As we mentioned above, the acoustic permeability equals the static one for \( s_p=0.423 \). For cylindrical pores, the pore shape factor ratio reaches its maximum of \( s_p=0.5 \) and the acoustic permeability is slightly lower than the static permeability. This is because the specific heat ratio is higher than one in air and the slow wave velocity depends on the isothermal sound velocity \( v_T=v_f/\gamma^{1/2} \) rather than on the adiabatic velocity \( v_f \), which is measured in an infinite medium. For pores with noncircular cross sections, the pore shape factor ratio \( s_p \) can be much lower than 0.5. For example, in the case of equilateral triangle cross section, \( s_p \) is as low as 0.158.16 We shall show that, in the case of the measured high-permeability sandstones, the pore shape factor ratio is between 0.2 and 0.3, therefore the acoustic permeability is somewhat higher than the static one.

Recently, Champoux and Allard extended the dynamic permeability model20 of Johnson et al. for air-saturated solids.21,22 This new model properly accounts for the fact that viscous effects are dominated by small pore cross sections and thermal effects by large ones by introducing an additional characteristic length parameter. However, the use of this more rigorous theory apparently would not improve the agreement with our experimental data at high frequencies where the only major discrepancy occurs. Therefore, we shall use Attenborough's simpler analytical technique15,16 to calculate the velocity and attenuation coefficient of slow compressional waves propagating in air-filled porous materials for comparison to our experimental data.

In order to demonstrate the main features of slow wave propagation in air-filled porous solids, let us calculate the propagation parameters and the complex acoustic impedance in cemented glass bead specimens of different grades. Table II lists the material parameters used in these calculations. The porosity (\( \phi=0.3 \)) and static permeability were estimated from the manufacturer's (Eaton Products International, Inc.) specifications. The high-frequency tortuosity was taken to be 1.79 as the most typical parameter obtained from acoustical and electrical measurements on similar materials.6,18,23,24 Finally, the pore shape factor ratios were chosen by matching the analytical results to our experimental data to be presented in the next section.

Figure 3 shows the normalized velocity \( v/v_f \) and the attenuation coefficient \( \alpha \) of the slow compressional wave in the air-filled porous samples listed in Table II as functions of frequency. In the diffuse regime, i.e., at low frequencies, both the velocity and the attenuation coefficient are proportional to the square-root of frequency:

\[
\alpha_n^{(\text{low})} = \frac{\alpha}{\gamma f} = \frac{\alpha}{\gamma f} = \frac{\tau_\alpha}{\gamma f} \approx 55 \text{ dB}.
\] (15)

In the propagating regime, i.e., at high frequencies, the velocity approaches a constant value while the attenuation coefficient is again proportional to the square-root of frequency, although the proportionality coefficient is slightly different from the low-frequency value (see Fig. 2):

\[
\alpha_n^{(\text{high})} = \frac{\tau_\alpha}{\gamma f} = 55 \text{ dB}.
\] (16)

The normalized attenuation coefficient \( \alpha_n \), i.e., the total attenuation over a single wavelength, is constant,

\[
\alpha_n^{(\text{low})} = \alpha^{(\text{low})} f = 2\pi(\text{Nepers}) \approx 55 \text{ dB}.
\] (15)

where the second term of Eq. (17) is approximately 1.46 for air. It is interesting to note that the ratio between the high- and low-frequency asymptotic values of the attenuation coefficients is
II. EXPERIMENTAL TECHNIQUE AND RESULTS

Figure 6 shows the block diagram of the experimental system used in this study. It is based on a recently developed method using the transmission of airborne ultrasonic waves through thin plates of air-filled porous specimens to investigate the propagation parameters of the slow compressional wave. Standard ultrasonic NDE equipment was used without any particular effort to obtain high generation or detection sensitivity. The rather poor coupling between the applied contact transducers and air resulted in a rather low, but fairly constant, sensitivity over a wide frequency range of 50-500 kHz. In certain cases, we replaced the ultrasonic transmitter and receiver by a com-

The simplest thing to do is to approximate the actual acoustic impedance by its real-valued high-frequency asymptote. From Eqs. (19) and (20),

\[ T_0^{(\text{high})} = \frac{4}{2 + \tau_{\infty}^{1/2} / \phi / \phi^{1/2}}. \]  

For example, \( T_0^{(\text{high})} \approx -4.5 \text{ dB} \) for \( \phi = 0.3 \) and \( \tau_{\infty} = 1.79 \). In this way, we inevitably underestimate the total transmission loss and, consequently, overestimate the attenuation coefficient. Also, because of the phase shift caused by the complex nature of \( T_0(\omega) \), we slightly underestimate the slow wave velocity in samples of small thickness. In order to get better agreement between the experimental measurements and theoretical calculations, we can easily correct our analytical results for the difference between the actual transmission loss \( T_0(\omega) \) and its real valued asymptote \( T_0^{(\text{high})} \). The measured transmission coefficient, \( T_{m}(\omega) \), can be expressed as

\[ T_{m}(\omega) = T_0^{(\text{high})} \exp(i\omega d / v_{a}) \exp(-\alpha_{a} d), \]

where \( d \) is the thickness of the specimen and \( v_{a} \) and \( \alpha_{a} \) are the apparent velocity and attenuation coefficient, which are corrected according to \( d \). Figure 5 shows the apparent velocity and attenuation coefficient of the slow compressional wave in air-filled cemented glass bead specimen grade 55 as functions of frequency for different sample thicknesses. Because of the larger impedance mismatch and the additional phase shift at lower frequencies, the apparent velocity drops while the apparent attenuation increases in thin samples. In the propagating regime (above approx. 60 kHz), the corrections are negligible.

In the diffuse regime, where transmission-type measurements are not feasible because of the very high normalized attenuation coefficient, we have to determine the complex acoustic impedance from reflection-type measurements. In the propagating regime, we can use transmission measurements to determine the velocity and attenuation coefficient of the slow compressional wave. Even then, but especially in the transition region between the diffuse and propagating regimes, we have to take into account the total (fluid/fluid-saturated porous solid/fluid) transmission loss \( T_0 \) caused by the significant acoustical impedance mismatch between the air and the air-filled specimen:

\[ T_0(\omega) = \frac{4}{2 + Z(\omega) / Z_f + Z_f / Z(\omega)}. \]  

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\[ Z(\omega) = \frac{2 + \tau_{\infty}^{1/2} / \phi / \phi^{1/2}}{2 + \tau_{\infty}^{1/2} / \phi / \phi^{1/2}}. \]

while the imaginary part diminishes.

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\[ Z_{\text{high}}^{(\text{high})} = \tau_{\infty}^{1/2} / \phi, \]

while the imaginary part diminishes.

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FIG. 5. (a) Apparent velocity and (b) attenuation coefficient of the slow compressional wave in air-filled cemented glass bead specimen grade 55 as functions of frequency for different sample thicknesses.

FIG. 6. Block diagram of the experimental system.

FIG. 7. Comparison between the theoretically predicted and experimentally measured slow wave velocities in cemented glass bead specimens for grades (a) 55 and (b) 90.

between the transducers were digitally stored. Then the computer selected the first five cycles of the signal, from which it determined the insertion loss $L_i$ and insertion delay $t_i$. In spite of the relatively narrow bandwidth of the tone-burst excitation, the frequency spectrum of the transmitted signal was appreciably different from that of the direct transmission because of the strong frequency-dependent attenuation of the specimen. The insertion loss was determined by Fourier transforming the gated signals and calculating the ratio between the two spectra at the maximum of the attenuated one. The insertion delay was determined by finding the maximum of the cross-correlation function of the two signals. The normalized velocity and apparent attenuation coefficient were calculated from the insertion delay $t_i$ and loss $L_i$ by the following simple equations:

$$\frac{v_a}{v_f} = \frac{1}{1 + t_i v_f / d}$$  \hspace{1cm} (24)

and

$$\alpha_a = \frac{(L_i - T_0)}{d}.$$  \hspace{1cm} (25)
The thicknesses of the specimens varied between 1 and 5 mm to accommodate different permeabilities over the widest possible frequency range. Because of the very high attenuation in these samples, resonance peaks in the transmission are generally very weak but sometimes, especially at lower frequencies, faintly visible. The samples were approximately 4 in. in diameter and, especially the rocks, showed significant inhomogeneity of the attenuation distribution within this area. The presented data include 50–250 measurements taken at randomly chosen positions as the frequency was scanned up and down within the usable range.

First, let us show some typical results on synthetic materials of well-defined pore structure to demonstrate the accuracy of the measurement. Later, we shall present similar results on different natural rocks of much more complicated pore structure to demonstrate the feasibility of ultrasonic evaluation of such less-permeable formations, too. The first series of experiments were conducted on cemented glass bead specimens previously listed in Table I. Generally, we found excellent consistency between the theoretically predicted and experimentally measured slow wave velocities. Figure 7 shows two examples of the comparison between theoretical and experimental results for grades 55 and 90. For the attenuation coefficient, the agreement is less perfect. Figure 8 shows the comparison between the theoretical and experimental results for grades 15 through 175. For the smallest pore size (grade 15), the agreement is still acceptable indicating that the total attenuation is dominated by viscous losses throughout the whole frequency range. As the pore size is gradually increased, viscous losses decrease while scattering losses become stronger. As we have shown above (see Table I), there is not a frequency-window where both viscous and scattering losses are simultaneously negligible in air-saturated porous materials. At first only at higher frequencies (grades 40 and 55) then throughout the whole frequency range (grades 90 and 175), the experimentally observed attenuation coefficient significantly overshoots the theoretical
TABLE III. Estimated material and geometrical parameters of natural rocks.

<table>
<thead>
<tr>
<th>Type</th>
<th>$\phi$</th>
<th>$\kappa_0$ $(10^{-12} \text{ m}^2)$</th>
<th>$\tau_c$</th>
<th>$s_p$</th>
<th>$d$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cavallo buff massillon</td>
<td>0.15</td>
<td>0.7</td>
<td>2.6</td>
<td>0.3</td>
<td>2.4</td>
</tr>
<tr>
<td>Sunset blush massillon</td>
<td>0.15</td>
<td>0.6</td>
<td>2.8</td>
<td>0.3</td>
<td>2.1</td>
</tr>
<tr>
<td>Copper-variegated sandstone</td>
<td>0.13</td>
<td>0.6</td>
<td>2.3</td>
<td>0.3</td>
<td>1.7</td>
</tr>
<tr>
<td>Berea sandstone</td>
<td>0.12</td>
<td>0.2</td>
<td>3.2</td>
<td>0.2</td>
<td>1.5</td>
</tr>
</tbody>
</table>

The primary purpose of our experiments on synthetic porous materials was to demonstrate the accuracy of the measuring system and to verify the feasibility of Attenborough's simple model for random but statistically well-defined permeable formations. Our next step was the adaptation of this technique to natural rocks of relatively high permeability between 100 and 1000 mDarcy. Basically, the results were fairly similar to those obtained for synthetic materials, although the scatter of the data became somewhat larger due to inherent macroscopic inhomogeneities found in most natural rocks. Table III lists the materials used in this part of the study as well as their relevant physical and geometrical properties. The pore shape factor ratio is between 0.2 and 0.3, i.e., significantly lower than for the glass bead specimens. With the exception of the permeability of the Berea sandstone specimen, which was measured at the Lawrence Livermore National Laboratory, all parameters were adjusted to obtain the best agreement between the analytical results and the experimental data. Later we plan to determine the three basic material parameters, namely the porosity, the permeability, and the tortuosity, by separate measurements and adjust only the pore shape factor ratio, which is the only truly independent acoustic parameter. Figures 9–12 show the normalized slow wave velocity and attenuation coefficient in different natural rocks. Again, the experimentally measured velocity is consistent with the analytical results while the attenuation coefficient exhibits higher-than-predicted values and more or less linear frequency dependence [the strange behavior of the calculated "apparent" attenuation coefficient at low frequencies is caused by the increasing contribution of the impedance mismatch in the total insertion loss, as it was shown in Fig. 5(b)].
III. DISCUSSION

Transmission of airborne ultrasonic waves through thin air-filled porous plates was used to study slow wave propagation in permeable solids. In the diffuse regime, i.e., at low frequencies, the velocity and the attenuation coefficient contain the same information on the permeable formation. The attenuation is linearly while the velocity is inversely proportional to the square root of $s^2/\kappa_0$. Since the static permeability $\kappa_0$ and the pore shape factor ratio $s_p$ always occur in the same combination through the normalized pore radius $\xi$, they cannot be separated by acoustical measurements. Instead, $\kappa_0/\xi^2$ was used to define a new material parameter, the so-called acoustic permeability $\kappa_a$, which can be determined from the low-frequency propagation parameters of the slow compressional wave. Comparison of the acoustic permeability $\kappa_a$ and the static permeability $\kappa_0$ could yield valuable information on the geometry of the pore space.

In the propagating regime, i.e., at high frequencies, only the velocity seems to be consistent with the theoretical predictions. Anyway, the velocity by itself is sufficient to determine the high-frequency tortuosity $\tau_\infty$ of the material while the attenuation coefficient would provide only redundant information on $s^2/\kappa_0 \phi_0$, which is better determined from the low-frequency behavior. The high-frequency attenuation contains valuable additional information on the geometry of the pore space and the surface roughness of the pore channels. Our experimental results on glass bead specimens indicate that the slope of the attenuation coefficient versus frequency curve approaches the same $\partial \alpha/\partial f \approx 0.033$ dB/mm kHz value at high frequency for all grades. These samples are cemented from spherical glass beads of different diameters and all have approximately the same porosity, tortuosity, and pore geometry (grade designations used to identify EP brand porous structures denote the maximum interstitial pore diameter in microns). Since the high-frequency asymptote of the slow wave velocity is $V(\text{high}) \approx 250$ m/s ($\kappa_{\text{high}} \approx 333$ m/s, $\tau_\infty \approx 1.79$), the normalized attenuation is constant at $\alpha_a \approx 8.3$ dB.

Since, at least in the upper part of the frequency range, the grain size is comparable to the wavelength, the observed excess attenuation could be caused by elastic scattering. For example, the diameter of the cemented glass beads in grade 175 is $d \approx 600$ µm, i.e., $kd > 1$ above 60 kHz. Unfortunately, at this point, there seems to be no theoretical prediction available in the literature for the scattering induced attenuation of the slow compressional wave in permeable solids. On the other hand, even without further calculations, we can postulate from the linear frequency dependence of the attenuation coefficient that it has to be independent of the characteristic dimension of the scatterer. This is because the dimensionality of the problem
requires that the normalized attenuation be independent of the size of the scatterer when it is independent of the wavelength. This conclusion is at least not contradicted by our experimental results in the self-similar glass bead specimens, where the grain size changed approximately one order of magnitude without any significant change in the high-frequency slope of the attenuation versus frequency curve. Of course, this apparent confirmation might be a pure coincidence and cannot prove by itself that the observed excess attenuation is caused by elastic scattering. Actually, it would be rather unusual if the scattering induced attenuation turned out to be independent of the scatterer’s size.

The excess attenuation could be also caused by increased viscous drag due to the rather uneven pore geometry or surface roughness. At very high frequencies, where the viscous skin depth $\delta$ is so much smaller than the pore radius $a_p$ that it becomes comparable to the pore-wall surface roughness $h$, the viscous drag sharply increases and the attenuation of the slow compressional wave increases with frequency much faster than the expected square-root relation. This is because the peaks of the surface profile protrude into a region where the fluid velocity is already significant thereby greatly increasing the friction between the solid frame and the moving fluid column. In a recent paper, Gist used Norris’ analytical technique for the dynamic permeability of porous formations to show that there is a transition frequency range where the normalized attenuation coefficient of the slow wave is roughly independent of frequency. Both below ($a_p/\delta$) and above ($\delta/h$) this transition range, the attenuation coefficient is proportional to the square root of frequency. In the transition frequency range ($h/\delta < a_p$), the flow pattern of the fluid is determined by the pore geometry, but the viscous friction is greatly increased by surface roughness. Although such conditions are more typical for water saturation, the same effect might contribute to the observed excess attenuation in air-filled materials, too. For example, Waetzmann and Wenke found more then 50 years ago that the attenuation coefficient of airborne guided waves in ducts with rough walls deviates from the theoretically predicted square-root frequency dependence and the excess attenuation is linearly proportional to frequency.

In the low-frequency diffuse regime, the normalized attenuation is constant at $2\pi$. At the transition between the diffuse and propagating regimes, the normalized attenuation starts to decrease and approaches an inverse square-root frequency dependence. At very high frequencies, where the viscous skin depth becomes negligible to the surface roughness, the normalized attenuation would again assume an inverse square-root frequency dependence but for the very high scattering losses. In liquid-saturated specimens, the attenuation coefficient can drop to a very low value before scattering losses become important. In air-saturated specimens, there is not such a wide window for more or less attenuation-free, dispersion-free slow wave propagation and the attenuation coefficient cannot significantly drop before scattering losses start to dominate. Consequently, the relative role of the surface roughness in-duced excess attenuation is much higher in water-saturated samples than in air-saturated ones. Although there is very little known about the actual frequency dependence of the slow wave attenuation in water-saturated porous solids, the predicted linear relationship seems to be in good agreement with previously published data by Plona and Winkler (see lines B and C in Fig. 4 of Ref. 24). In air-saturated samples, the surface roughness induced excess attenuation is more difficult to separate from scattering losses which also start to give significant contributions to the observed total attenuation immediately above the diffuse regime. Obviously, further analytical efforts are needed to develop appropriate models for these attenuation mechanisms so that the measured data can be evaluated in terms of microscopic geometrical properties of the permeable formation.

IV. CONCLUSIONS

The propagation of slow compressional waves in air-saturated permeable solids was studied by experimental means between 10 and 500 kHz. Due to the excellent sensitivity of the suggested experimental technique, low-permeability materials including natural rocks can be inspected, too. Currently, the threshold sensitivity of our system is approximately 100 mDarcy. In the low-frequency (diffuse) regime, the experimental results are consistent with theoretical predictions. The phase velocity and attenuation coefficient are determined by the “acoustic” permeability of the specimen and both increase proportionally to the square root of frequency. In the high-frequency (propagating) regime, the experimental results are consistent with the theoretical predictions for the phase velocity but not for the attenuation coefficient. The phase velocity asymptotically approaches a maximum value determined by the tortuosity of the specimen while the attenuation coefficient becomes linearly proportional to frequency instead of the expected square-root relationship. It was suggested that the observed discrepancy is partly due to irregular pore geometry and partly to pore-wall roughness, which substantially reduce the high-frequency dynamic permeability of the specimens. In addition, scattering losses also give significant contributions to the observed total attenuation, since, for air saturation, there is no frequency “window” where both viscous and scattering losses are weak simultaneously. Further theoretical efforts are needed to incorporate these excess attenuation mechanisms into existing analytical models.

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