Role of anisotropy in noncontacting thermoelectric materials characterization

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Inclusions and other types of imperfections in metals can be nondestructively detected by noncontacting magnetic measurements that sense the thermoelectric currents that appear when the specimen is subjected to directional heating and cooling. The detectability of small imperfections is ultimately limited by the intrinsic thermoelectric anisotropy and inhomogeneity of the material to be inspected. This article presents an analytical method for calculating the magnetic field produced by thermoelectric currents in anisotropic materials under two-dimensional directional heating and cooling. Experimental results from a textured Ti–6Al–4V titanium-alloy plate are shown to be in very good agreement with the predictions of this model. The described analytical method can be used to optimize thermoelectric inspection procedures and to evaluate the macroscopic texture of metals from their characteristic magnetic signatures. © 2002 American Institute of Physics.

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I. INTRODUCTION

Thermoelectricity is caused by coupled transport of heat and electricity in metals, which leads to a number of interesting phenomena, some of which can be exploited for nondestructive evaluation (NDE) and materials characterization. Most existing thermoelectric NDE methods are based on the well-known Seebeck effect that is commonly used in thermocouples to measure temperature at the junction between two different conductors. When two junctions between different metals forming an electrical circuit are kept at different temperatures, an electromotive force is produced. Ideally, regardless of how high the temperature difference between the junctions is, only thermocouples made of different materials, or more precisely, materials of different thermoelectric power, will generate a thermoelectric signal. This unique feature makes the simple thermoelectric tester one of the most sensitive material discriminators used in NDE. The thermoelectric power of metals is sensitive to a variety of material properties that can affect the measurement. Clearly, chemical composition exerts the strongest effect on the thermoelectric properties and accordingly the basic application of thermoelectric materials characterization is metal sorting. However, it is well known that under special conditions materials of identical chemical composition can also produce an efficient thermocouple as a result of different heat treatment, hardening, texture, residual stress, fatigue, etc., which can be further exploited for nondestructive testing of materials.

It was recently demonstrated by Hinken and Tavrin and by Maslov and Kimra that self-referencing thermoelectric measurements can be done in an entirely noncontacting way by using high-sensitivity magnetic detectors to sense the weak thermoelectric currents around inclusions and other types of inhomogeneities when the specimen to be tested is subjected to directional heating or cooling. A schematic diagram of the self-referencing thermoelectric method with noncontacting magnetic sensing is shown in Fig. 1. External heating or cooling is applied to the specimen to produce a modest temperature gradient (∼1–10 °C/cm) in the region to be tested. As a result, different points on the boundary between the host material and the inclusion will be at different temperatures, therefore, also at different thermoelectric potentials. These potential differences will drive opposite thermoelectric currents inside and outside the inclusion. The thermoelectric currents form two local loops that run in opposite directions on the two sides of the inclusion relative to the prevailing heat flux, which can be detected by scanning the specimen with a sensitive magnetometer. Since the surrounding intact material serves as the “reference” electrode and there is no artificial interface between the host and the imperfect region to be detected, the detection sensitivity to variations in material properties could be very high and subtle effects such as local plastic deformation can also be detected.

In a recent article, Nagy and Nayfeh developed an analytical model to predict the magnetic field produced by thermoelectric currents around surface-breaking and subsurface spherical inclusions in a homogeneous host material under external thermal excitation. These predictions were subsequently experimentally verified using surface-breaking spherical tin inclusions of varying diameter in copper. These experiments also revealed that the detectability of small inclusions and subtle imperfections by this method is ultimately limited by the intrinsic thermoelectric anisotropy and inhomogeneity of the material to be inspected. The probability of detection (POD) of a given material flaw is determined by the resulting signal-to-noise ratio rather than by the absolute magnitude of the signal itself. With the exceptions of a few special cases, most sensors used in NDE are not limited by temporally incoherent electrical noise but rather by tem-
porally coherent material noise that cannot be eliminated by
time averaging. Strictly speaking, material “noise” is really
unwanted background “signal” that is called noise only be-
cause it interferes with, and often conceals, the flaw signal to
be detected. Such a material signature is produced by micro-
structural features (e.g., grains, second phases, or precipita-
tions) and macrostructural features (e.g., heat-affected, work-
hardened, strained, and textured regions).

Let us consider an example of how this material signature
limits the detectability of weak imperfections by the
thermoelectric method. The noncontacting thermoelectric
method has been recently adapted to the characterization of
weak surface and near-surface imperfections caused by lo-
calized plastic deformation that is usually very difficult to
detect by conventional NDE methods because they are effec-
tively hidden by the accompanying surface roughness. The
same approach can also be adopted to the characterization of
near-surface material damage on fretted Ti–6Al–4V speci-
mens. As an example, Fig. 2 shows magnetic scans of two
Ti–6Al–4V specimens taken at a $\nabla T \approx 13 \, ^\circ \text{C/cm}$ temperature
gradient for two opposite heating directions. The speci-
men without fretting damage exhibits a background signature
of $\approx 7$ nT peak-to-peak amplitude that is caused by the in-
trinsic anisotropic texture. After substantial fretting damage
was induced over a spot of $\approx 1/2$ in. diam at the center (as
indicated schematically by the superimposed circle), the
magnetic signature essentially doubled to $\approx 14$ nT. The main
advantage of the thermoelectric method is that it is sensitive
only to the “material” effects of the damage, namely, re-
idual stress, local texture, and increased dislocation density,
but it is entirely insensitive to its “geometrical” by-product,
i.e., the rough surface topography. However, the detectabil-
ity of weak imperfections is obviously adversely affected by
the presence of the rather strange looking background sig-
nature in the intact material, that can be reduced only by an-
nealing above the recrystallization temperature, which would
also completely change the existing microstructure.

The appearance of the above-described background sig-
nature in noncontacting thermoelectric inspection is not sur-
prising at all. For example, it has been known for a long time
that pure cold-rolled metals can exhibit a rather significant
variation of $\approx 1\%$ in thermoelectric power with respect to the
same metal in the annealed condition. Texture-induced an-

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{Schematic diagram of noncontacting detection of material imperfec-
tions by magnetic monitoring of thermoelectric currents.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure2.png}
\caption{Magnetic scans of a Ti–6Al–4V specimen before and after fretting
($\nabla T \approx 13 \, ^\circ \text{C/cm}$ temperature gradient, 3 in.$\times$3 in.).}
\end{figure}

The only approach that pure cold-rolled metals can exhibit a rather significant
variation of $\approx 1\%$ in thermoelectric power with respect to the
same metal in the annealed condition. Texture-induced an-
isotropy can also lead to significant thermoelectric signals
between regions of different orientation. Generally, the
anisotropy is partly due to partial alignment of the crystallo-
graphic orientations of the neighboring grains and partly to
morphological features such as preferred orientation of elong-
gated grain boundaries, dislocations, slip bands, etc. It
should be mentioned that, in contrast to mechanical proper-
ties, thermoelectric properties of cubic materials (aluminum,
copper, nickel, steel, etc.) do not exhibit crystallographic an-
isotropy. The few structural metals of great practical impor-
tance that preferentially crystallize in noncubic (hexagonal)
symmetry, therefore, do exhibit thermoelectric anisotropy,
are titanium and its alloys. Another important effect that
might contribute to the thermoelectric material signature is
the substantial stress dependence of the thermoelectric
power, that could result in significant point-to-point varia-
tions as well as orientation dependence when residual or ex-
ternally induced stresses are present in the material.

Our present goal is to study how thermoelectric aniso-
ropy causes the characteristic background signature shown in
Fig. 2(a) in order to reliably estimate the POD of certain
material flaws. From this point of view, all the above-
mentioned effects (i.e., hardening, texture, residual stresses)
are clearly negative as far as they adversely affect the detect-
ability of flaws. On the other hand, it is also clear that the
unique sensitivity of the thermoelectric method to these ef-
ects can be readily exploited in the future for the detection
and quantitative characterization of these subtle material
variations.

II. ANALYTICAL MODEL

The primary goal of the following analytical effort is to
predict the magnetic background signature in homogeneous
anisotropic media by using the classical coupled transfer
theory of thermoelectricity. Thermoelectricity is a result of intrinsically coupled transport of electricity and heat in metals, which can be expressed by the following constitutive relationship:  

\[
\mathbf{j} = \begin{bmatrix} \sigma & \epsilon \\ \bar{\epsilon} & \kappa \end{bmatrix} \begin{bmatrix} -\nabla \Phi \\ -\nabla T \end{bmatrix},
\]

(1)

where \( \mathbf{j} \) is the electrical current density, \( \mathbf{h} \) is the thermal flux, \( \Phi \) is the electric potential, \( T \) is the temperature, \( \sigma \) denotes the electrical conductivity measured at uniform temperature, \( \kappa \) is the thermal conductivity for zero electrical field, and \( \epsilon \) and \( \bar{\epsilon} \) are thermoelectric coupling coefficients that can be expressed by the absolute thermoelectric power \( S \) of the material as \( \epsilon = \sigma S \) and \( \bar{\epsilon} = \sigma ST \). The thermal conductivity for zero electrical field \( \kappa \) can be easily expressed by the thermal conductivity of the material for zero electrical current, \( k \), which is often easier to determine experimentally, as \( \kappa = k + \sigma S^2 T \). The difference between these two thermal conductivities is due to the thermoelectric coupling in the material. We can write that \( \kappa = k(1 + \eta) \), where \( \eta = \sigma S^2 T / k \) is a dimensionless factor that provides a measure of the strength of coupling between thermal and electrical transports. For typical metals, the coupling factor is relatively small somewhere between \( 10^{-3} \) and \( 10^{-2} \), an important fact that will be exploited in our following calculations.

The total energy flux \( \mathbf{h} + \nabla \Phi \mathbf{j} \) includes the thermal flux plus and additional term representing the changing electric potential of the electrons. The rate at which heat is evolved, per unit volume, at any point in the material is \( \nabla \cdot \mathbf{h} + \nabla \Phi \cdot \mathbf{j} \), where we exploited Maxwell’s law that \( \nabla \cdot \mathbf{j} = 0 \). If we now assume the coupling coefficients \( \epsilon \) and \( \bar{\epsilon} \) to be small such that their individual squares and products can be neglected, then for thermal loading we can neglect \( \nabla \Phi \cdot \mathbf{j} \) and, for the steady state, we can conclude that \( \nabla \cdot \mathbf{h} = 0 \). Imposing the conditions that the divergences of both the electrical current density and the thermal flux vanish and noting that \( \sigma \kappa - \epsilon \bar{\epsilon} \neq 0 \), Eq. (1) requires that the Laplacians of \( T \) and \( \Phi \) vanish individually, i.e.,

\[
\nabla^2 T = 0 \quad \text{and} \quad \nabla^2 \Phi = 0.
\]

(2)

For a homogeneous isotropic medium, \( \sigma, \kappa, \) and \( S \) are scalar quantities that do not depend directly on the spatial coordinates, though generally they do depend on temperature, which can introduce an indirect spatial variation. For purely thermal excitation in an isotropic medium, the local electrical field \( \mathbf{E} = -\nabla \Phi \) is strictly parallel to the temperature gradient \( \nabla T \) and the electrical current density can be shown to identically vanish everywhere. This means that regardless of the size, shape, and material properties of a homogeneous isotropic specimen, no thermoelectric current will be generated by any type of heating or cooling. In an anisotropic medium, \( \sigma, \kappa, \) and \( S \) are second-order tensor quantities and the local electrical field is not necessarily parallel to the temperature gradient. Though the divergence of the electrical current density \( \nabla \cdot \mathbf{j} = 0 \) is still identically zero everywhere by virtue of Maxwell’s theorem, the curl of the electrical current density \( \nabla \times \mathbf{j} \neq 0 \) does not necessarily vanish. Therefore, specimens made an anisotropic materials can produce a nonvanishing thermoelectric current distribution, and an associated nonvanishing magnetic field, even when the material is completely homogeneous. The nonvanishing thermoelectric current distribution in anisotropic media is caused by nonuniform heat flow that can be due to either nonuniform heating and cooling or the irregular shape of the specimen, e.g., a rivet hole that forces the heat flux to go around it.

A. Infinite line source along a principal direction

In order to demonstrate how anisotropy can induce a nonvanishing thermoelectric current distribution in a specimen under forced heating and cooling, let us consider an infinite line source acting in an infinite anisotropic medium. Since all material properties are affected by the same microstructural symmetry, we are going to assume that electrical conductivity \( \sigma \), thermal conductivity \( \kappa \), and thermoelectric power \( S \) tensors all exhibit the same principal directions, though their degree of anisotropy might be very different. In order to further simplify the calculations, we are going to assume that the line source is oriented along one of the principal directions of the medium, which is aligned with the \( x_3 \) axis of a Cartesian (principal material) coordinate system. In this way, the problem becomes essentially two-dimensional with both the heat flux and the thermoelectric current limited to the \( x_1, x_2 \) plane of symmetry. In order to satisfy our two field equations, \( \nabla \cdot \mathbf{j} = 0 \) and \( \nabla \cdot \mathbf{h} = 0 \), we seek solutions in the following forms:

\[
T = B \ln \left[ \frac{x_1^2}{\beta_1} + \frac{x_2^2}{\beta_2} \right] \quad \text{and} \quad \Phi = D \ln \left[ \frac{x_1^2}{\beta_1} + \frac{x_2^2}{\beta_2} \right].
\]

(3)

Here, \( B \) and \( D \) are currently unknown amplitudes that will be later determined from the appropriate boundary conditions and \( \beta_1 \) and \( \beta_2 \) are diffusion coefficients determined by the anisotropic material properties of the medium. The characteristic equation for \( \beta_1 \) and \( \beta_2 \) can be obtained from the field equations and the constitutive relations given in Eq. (1) as follows:

\[
\begin{bmatrix}
\frac{\epsilon_1}{\beta_1} & \frac{\sigma_1}{\beta_2} & \frac{\kappa_1}{\beta_1} & \frac{\epsilon_1}{\beta_2} \\
\frac{\sigma_2}{\beta_1} & \frac{\epsilon_2}{\beta_2} & \frac{\kappa_2}{\beta_1} & \frac{\epsilon_2}{\beta_2} \\
\frac{\kappa_1}{\beta_1} & \frac{\kappa_2}{\beta_2} & \frac{\epsilon_1}{\beta_1} & \frac{\epsilon_2}{\beta_2} \\
\frac{\kappa_1}{\beta_1} & \frac{\kappa_2}{\beta_2} & \frac{\sigma_1}{\beta_2} & \frac{\sigma_2}{\beta_2}
\end{bmatrix}
\begin{bmatrix}
B_1 \\
B_2 \\
D_1 \\
D_2
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix}.
\]

(4)

Nontrivial solutions are found by first solving the eigenvalue problem and then determining the corresponding eigenvectors. For brevity, we are going to introduce the following notation for the ratios of the diffusion coefficients and material properties: \( \beta = \beta_1 / \beta_2 \), \( \sigma = \sigma_1 / \sigma_2 \), \( \epsilon = \epsilon_1 / \epsilon_2 \), and \( \kappa = \kappa_1 / \kappa_2 \). Furthermore, \( \eta = (\epsilon_1 \epsilon_2) / (\sigma_1 \kappa_2) = \sigma_2 S^2 T / \kappa_2 \) will be used without a subscript to denote the thermoelectric coupling of the material in the second principal direction. The sought characteristic equation for \( \beta \) is obtained by requiring that the determinant of the characteristic matrix in Eq. (4) vanish

\[
\eta(\epsilon - \beta)^2 - (\sigma - \beta)(\kappa - \beta) = 0.
\]

(5)

This quadratic equation yields two eigenvalues that are denoted by \( \beta_p \) and \( \beta_h \):
\[ \beta_{p,n} = \frac{\sigma + \kappa - 2 \epsilon \eta \pm \sqrt{(\sigma - \kappa)^2 - 4 \eta (\sigma - \epsilon)(\epsilon - \kappa)}}{2(1 - \eta)}, \]

where subscripts \( p \) and \( n \) correspond to the positive and negative values of the square root in the numerator. Finally, the corresponding eigenvectors can be determined from either rows of Eq. (4) as follows:

\[ \delta_p = D_p B_p = -\frac{\epsilon_2 (\epsilon - \beta_{p,n})}{\sigma^2 (\sigma - \beta_{p,n})} \quad \text{and} \quad \delta_n = D_n B_n = -\frac{\epsilon_2 (\epsilon - \beta_{n,n})}{\sigma^2 (\sigma - \beta_{n,n})} \]

Formal solution for the electrical current density and heat flux can be obtained from these temperature and electrical potential distributions by substituting them into the constitutive equation previously given in Eq. (1). After some algebraic manipulation, the electrical current density and thermal flux vectors are obtained as

\[ j = -2 \left( \frac{B_p \gamma_p}{\beta_p} + \frac{B_n \gamma_n}{\beta_n} \right) (x_1 e_1 + x_2 e_2), \]

and

\[ h = -2 \left( \frac{B_p \nu_p}{\beta_p} + \frac{B_n \nu_n}{\beta_n} \right) (x_1 e_1 + x_2 e_2), \]

where \( e_1 \) and \( e_2 \) are unit vectors along the \( x_1 \) and \( x_2 \) coordinate axes, respectively, and

\[ \gamma_{p,n} = \frac{\sigma - \epsilon}{\sigma - \beta_{p,n}} \quad \text{and} \quad \nu_{p,n} = \kappa_2 \left( 1 - \frac{\epsilon - \beta_{p,n}}{\sigma - \beta_{p,n}} \right). \]

The temperature and electric potential distributions can be written by extending Eq. (3) as follows:

\[ T = B_p \ln \left[ \frac{x_1^2 + x_2^2}{\beta_p} \right] + B_n \ln \left[ \frac{x_1^2 + x_2^2}{\beta_n} \right], \]

and

\[ \Phi = B_p \delta_p \ln \left[ \frac{x_1^2 + x_2^2}{\beta_p} \right] + B_n \delta_n \ln \left[ \frac{x_1^2 + x_2^2}{\beta_n} \right]. \]

Equations (8)–(12) are general solutions of the coupled field equations under the assumption that the total heat evolving per unit volume can be approximated by the divergence of the heat flux, i.e., thermoelectric contributions (Peltier and Thomson heat) as well as the Joule heat produced by the weak thermoelectric current are negligible. The only remaining unknowns, \( B_p \) and \( B_n \), can be readily determined from the boundary conditions prevailing around the line source at \( x_3 \). By integrating the electrical current density and heat flux, we get

\[ -4 \pi \gamma_p \sqrt{\beta_p} B_p - 4 \pi \gamma_n \sqrt{\beta_n} B_n = 0, \]

and

\[ -4 \pi \nu_p \sqrt{\beta_p} B_p - 4 \pi \nu_n \sqrt{\beta_n} B_n = \dot{Q}, \]

respectively, where \( \dot{Q} \) denotes the total heat power emanating from a unit length of the line source. These equations can be solved for \( B_n \) and \( B_p \) as follows:

\[ B_p = \frac{\dot{Q}}{4 \pi \sqrt{\beta_p} \gamma_p \nu_n - \nu_p} \quad \text{and} \quad B_n = \frac{\dot{Q}}{4 \pi \sqrt{\beta_n} \gamma_n \nu_p - \nu_n}. \]

Finally, the sought magnetic field can be obtained from Maxwell’s equation of \( \nabla \times \mathbf{H} = \mathbf{j} \) by integration. The only nonvanishing component is parallel to the line source

\[ H_z = \frac{\dot{Q}}{2 \pi} \frac{\tan^{-1} \left( \frac{\gamma_p x_1}{\sqrt{\beta_p}} \right) - \tan^{-1} \left( \frac{\gamma_n x_2}{\sqrt{\beta_n}} \right)}{\gamma_p \gamma_n}. \]

It is clear that there is no magnetic field along any of the principal directions \( (x_1 = 0 \text{ or } x_2 = 0) \), i.e., where the material appears to be “quasi-isotropic” in the sense that the corresponding components of the electrical current density and heat flux are parallel to the gradients of the temperature and electrical potential.

It should be mentioned that the degree of anisotropy in thermal \( \kappa \), electrical \( \sigma \), and thermoelectric \( \epsilon \) properties is much smaller than the corresponding degree of anisotropy in mechanical properties. First of all, crystallographic anisotropy is limited to noncubic materials only, which effectively excludes all of the important structural metals such as aluminum, steel, nickel, copper, etc. Even in noncubic metals like titanium, which preferentially crystallizes in hexagonal symmetry, the crystallographic anisotropy is only a few percent. In most cases, anisotropy is caused by relatively weak morphological features of texture, e.g., the evolution of elongated grain shapes in the direction of plastic deformation during cold working. Although, the thermal and electrical properties are perceptibly affected by such texturing in the material, the resulting degree of anisotropy is usually rather modest. Therefore, a very useful approximation can be made by assuming that the degree of anisotropy is small (0.9 < \( \sigma \), \( \kappa \), \( \epsilon < 1.1 \)), so that both \( \beta_p \) and \( \beta_n \) are close to unity and, consequently, Eq. (16) can be approximated by

\[ H_z \approx \frac{\dot{Q}}{2 \pi} \frac{\gamma_p x_1 x_2}{\sqrt{\beta_p} x_1^2 + x_2^2} = H_0 F(x_1, x_2), \]

which allows a much clearer separation of the parameters affecting the strength of the magnetic field, \( H_0 \), and a simplified universal spatial distribution

\[ F(x_1, x_2) = \frac{x_1 x_2}{x_1^2 + x_2^2}. \]

\( H_0 \) is a fairly complicated function of the material properties via Eqs. (6) and (10), but it does vanish when the degree of anisotropy diminishes, i.e., when \( \sigma \), \( \kappa \), and \( \epsilon \) all approach...
unity. The magnetic field depends only on the angular coordinate of the point of observation with respect to the principal directions but not on the distance from the source. In contrast to the electrical current density and heat flux, which are inversely proportional to the distance from the source, the magnetic field produced by an infinite line source exhibits no loss associated with the spreading of the heat. It should be mentioned again that $F(x_1, x_2)$ vanishes on the principal axes ($x_1 = 0$ or $x_2 = 0$) where there is no skewing between $\nabla \Phi$ and $\nabla T$, on one side and $j$ and $h$ on the other side.

**B. Infinite dipole along a principal direction**

In order to illustrate the main features of our analytical predictions for the anisotropic thermoelectric effect, we consider the case of a dipole consisting of a line source and a sink separated by a unit distance $b=1$, as is shown in Fig. 3. Because of the cancellation effect between the source and the sink, far away from the dipole the magnetic field decreases as the inverse of the distance from the dipole. Now, the principal directions of the anisotropic material ($x_1, x_2$) are rotated by an arbitrary angle of $\theta$ with respect to the dipole orientation along the $x_3 = \xi_3$ axis. Using superposition, the magnetic field of the dipole can be written in the physical coordinate system ($\xi_1, \xi_2, \xi_3$) as $H_z = H_0 F_d(\xi_1, \xi_2)$, where

$$F_d(\xi_1, \xi_2) = F_d (\xi_1, \xi_2 + 0.5) - F_d (\xi_1, \xi_2 - 0.5).$$

(19)

Here, $F_d(\xi_1, \xi_2)$ denotes the rotated distribution function of a line source, which can be easily calculated from the previously determined approximate spatial distribution function $F(x_1, x_2)$ by rotation from the material coordinate system ($x_1, x_2$) into the physical coordinate system ($\xi_1, \xi_2$), i.e., by substituting $x_1 = \xi_1 \cos \theta + \xi_2 \sin \theta$ and $x_2 = -\xi_1 \sin \theta + \xi_2 \cos \theta$. For simplicity, in Eq. (19) we normalized all coordinates to the separation distance $b$ between the source and the sink. Figures 4(a) and 4(b) show the normalized magnetic-field distributions $F_d(\xi_1, \xi_2)$ of the dipole shown in Fig. 3 for $\theta=0^\circ$ and $\theta=45^\circ$, respectively. When the dipole is aligned with the principal directions of the material ($\theta = 0^\circ$), two large asymmetric lobes of opposite signs appear on the two sides of the dipole direction. Since the main heat flux is along the dipole direction from the source towards the sink, this type of material signature is similar to the lobes produced by inclusions. When the dipole is oriented along the bisector between the principal directions of the material ($\theta=45^\circ$), both main lobes split into twin peaks and the distribution becomes symmetric with respect to the dipole direction. This type of material signature is rather unique and quite different from the typical pattern produced by inclusions. As we mentioned earlier, far away from the dipole the source and the sink increasingly cancel each other and the magnetic field is inversely proportional to distance.

**III. EXPERIMENTAL VERIFICATION**

In order to qualitatively verify the previously described analytical model, we conducted a series of experiments on a 1.9-mm-thick cold-rolled Ti–6Al–4V plate. First, we determined the principal material directions in the plane of the plate by shear-wave birefringence measurements in the through-thickness direction. The $x_1$ and $x_2$ principal directions were chosen to coincide with the polarization directions of the “fast” and “slow” shear waves, respectively, which were found to differ by $\approx 4.6\%$ on the average over the surface of the 305-mm-diam circular disk. Next, we used an elliptical eddy current coil of a 5:1 aspect ratio to determine the principal directions and anisotropy factor of the electrical conductivity in the plate between 30 and 300 kHz. We found that $\sigma = \sigma_1 / \sigma_2 \approx 1.023$, i.e., the degree of anisotropy was approximately 2.3% (the direction of high electrical conduc-

![FIG. 3. Schematic diagram of a dipole consisting of a line source and a sink separated by a unit distance $b=1$.](image-url)
tivity was parallel to the polarization direction of the fast through-thickness shear wave. The thermoelectric anisotropy was measured using a calibrated ATS-6044 Alloy ThermoSorter (Walker Scientific, Inc.). We found that the absolute thermoelectric power changed from $-5.01$ to $-4.86 \mu V/°C$ with orientation in the plane of the plate so that $S_1 / S_2 \approx 1.031$ (in our notation, then $\epsilon = \epsilon_1 / \epsilon_2 \approx 1.055$). Our limited goal is simply to qualitatively verify our theoretical model by comparing the rather strange spatial distributions of the analytical predictions and experimental observations. Therefore, the less easily measurable anisotropy factor of the thermal conductivity was not determined at this point.

Figure 5 shows a schematic diagram of the experimental arrangement used to map the spatial distribution of the magnetic signature. Numerous steps were taken to separate the sought magnetic signal of truly thermoelectric origin from potentially much stronger spurious artifacts (these measures are discussed in great detail in Ref. 9). The experimental data presented in this article are actually the difference between measurements taken at opposite heating directions and then divided by 2. As is indicated in Fig. 5, alternating the heating and cooling directions was achieved simply by changing the hot and cold water supplies connected to the heat exchangers and waiting a few minutes until steady-state conditions were reached. In order to improve heat conduction from the 12-mm-diam copper conductor rods to the specimen, a heat conducting silicone compound was applied between them. The centers of the heating and cooling spots was 50 mm apart and the temperature there was kept constant at 50 and 10 °C, respectively. Because of its particular importance in inter-

FIG. 6. Comparison between the analytical (left) and experimental (right) results for the two-dimensional distribution of the magnetic signature produced by an anisotropic material at different angles of $\theta$. The measured peak magnetic flux density is also listed for the experimental results (203 mm×203 mm scan, 2 mm lift-off), while in the case of the analytical results the peak is always unity due to normalization.
preparing the measured data, we should mention that, in order to eliminate the strong magnetic field of the Earth, we used AC coupling with a high-pass filter of very low cutoff frequency at 0.01 Hz. The pseudodynamic magnetic signals required for ac detection were produced by laterally (normal to the main heat flux) scanning the specimen at a speed of ∼20 mm/s. In addition to the relatively fast lateral scanning in the “line” direction, we also scanned the specimens at a much lower rate in the axial “frame” direction, i.e., parallel to the main heat flux. In this way, a 203 mm × 203 mm scan of 200 × 200 grid points took about 45 min. Whenever the magnetic field is asymmetric to the direction of heating, the resulting bipolar line signal does not exhibit a significant DC component and will be recorded without substantial distortion. However, if the magnetic field is symmetric to the direction of heating, the resulting unipolar line signal loses its DC component and will be significantly distorted.

Figure 6 shows the comparison between the analytical and experimental results for the two-dimensional distribution of the magnetic signature produced by an anisotropic material at different angles of θ. In the case of the experimental results on the right the measured peak magnetic flux density is also listed, while in the case of the analytical results on the left the peak is always unity. Whenever θ is close to either 0° or 90°, i.e., the principal material directions are aligned with the heating/cooling direction, two large asymmetric lobes of opposite sign appear on the two sides and the resulting line scans are bipolar in nature and are, therefore, well reproduced by the experimental data. However, when the dipole is aligned along the bisectors between the principal directions of the material (θ = 45°), both main lobes split into twin peaks and the distribution becomes symmetric with respect to the dipole direction. As we indicated above, these distributions are inherently distorted by the necessity of using AC coupling. This distortion effectively eliminates the average signal in each line scan thereby producing virtual peaks and valleys of opposite sign with respect to the dominating principal features. As a result, the measured distributions exhibit additional secondary bumps not predicted by the theory. In spite of this distortion, the good correlation of the experimental data with the analytical predictions is still quite obvious.

Finally, it should be mentioned that the spatial resolution of the experimental images is inherently lower than that of the theoretical predictions. This effect is only partially due to the approximately 3 mm diam of the flux gate sensor itself and it is mainly caused by the finite thickness of the plate and the significant ∼12 mm effective lift-off of the sensors. Except for the close vicinity of the one-sided heating and cooling areas, the heat flux and electrical current distribution within the thin plate are essentially the same as in the infinite medium assumed in our analytical model. However, the magnetic field produced by the thermoelectric currents in a plate of finite thickness should be calculated by using the Biot–Savart integration technique that leads to a loss of lateral resolution comparable to the lift-off distance, which is actually much larger than the diameter of the sensor. It is probable that the observed drop in the measured peak magnetic flux density at around θ = 45° is mainly caused by this insufficient resolution to capture the sharp twin peaks that occur at this orientation. Considering the inherently lower resolution and AC distortion of the experimental images, the agreement with our theoretical predictions is very good and clearly indicates that the suggested analytical model truthfully captures the main features of the anisotropic magnetic signature of textured materials.

IV. CONCLUSIONS

This article presented an analytical method for calculating the magnetic field produced by thermoelectric currents in anisotropic materials under two-dimensional directional heating and cooling. These results clearly indicate that the earlier observed strange background signatures in textured specimens can be attributed to the thermoelectric anisotropy of the material. In such specimens the best flaw detectability can be achieved by rotating the heating/cooling direction so that the anisotropic effect averages out. Only noncubic materials such as titanium alloys exhibit crystallographic anisotropy, though a much weaker morphological anisotropy can be also exhibited by textured cubic materials. Experimental results from a textured Ti–6Al–4V titanium-alloy plate were shown to be in very good qualitative agreement with the predictions of our analytical model. The results of this study can be used to optimize thermoelectric inspection procedures. Furthermore, they also indicate that noncontacting thermoelectric inspection can be used to characterize the macroscopic texture of materials by evaluating their magnetic signatures under external heating and cooling.

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References


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