Polynomial Algebra

Let \( f(x) = 5x^2 - 7x^2 + 9x + 11 \)
\( g(x) = 6x^2 + 3 \)

- **Matlab representation:**
  \[
  f = [5, -7, 9, 11] \quad \text{cubic}
  \]
  \[
  g = [6, 0, 3] \quad \text{quadratic}
  \]
  \[
  \Rightarrow g = [0, g] \quad \text{cubic, highest coefficient zero}
  \]

- \( f \pm g = [5\pm0, -7\pm6, 9\pm0, 11\pm3] \)
  \[
  f \pm g = [5, -1, 9, 14, 8]
  \]
  \[
  \Rightarrow f+g = 5x^2 - x^2 + 9x + 14
  \]

- \( \text{conv}(f, g) = \text{product of 2 polynomials} \)

- \( [q, r] = \text{deconv}(\text{num}, \text{den}) = \text{division f/g} \)
  \[
  q = \text{quotient}
  \]
  \[
  r = \text{remainder}
  \]
  Note: For addition and subtraction, the polynomials must be of the same order.
  For multiplication or division, the order can be different.

- **Evaluation of a polynomial**
  \[
  \text{polyval}(f, x) \text{ or } \text{polyval}(g, x)
  \]
  \[
  \text{if } x = [0, .5, 10] \quad \text{array of values of } x
  \]
  \[
  \text{polyval}(f, x) \text{ will compute the polynomial at each value of } x \text{ of that array}
  \]

- **Roots of a polynomial equation, } f(x) = 0 \)
  \[
  rt = \text{root}(f)
  \]

**Example**

The following equation appears in structural vibrations:
\[
(\alpha-f^2)[(2\alpha-f^2)(\alpha-x)] + \alpha^2 f^2 - 2\alpha^3 = 0
\]
where \( \alpha \) is the natural frequency \( k/(4\pi^2 m) \)
\[
k = \text{spring constant}
\]
\[
m = \text{mass}
\]
Clearly this equation can be treated in two different ways: (i) a cubic in \( f^2 \), (ii) a 6\textsuperscript{th} order equation. No matter how it is coded, the solution will be the same.

- A cubic in \( f^2 = x \Rightarrow f = \pm \sqrt{x} \)
  \[
  P_1 = \alpha - x
  \]
  \[
  P_2 = 2\alpha - x
  \]
  \[
  P_3 = \alpha^2 x - 2\alpha^3
  \]
  \[
  \Rightarrow P_1[P_2 - \alpha^2] + P_3 = 0
  \]
**Matlab code 1**: \( \alpha = k/(4\pi^2 m) \)
- \( P_1 = [-1, \alpha] \)
- \( P_2 = [-1, 2\alpha] \)
- \( P_3 = [\alpha^2, -2\alpha^3] \)
- \( P_4 = \text{conv}(P_2, P_2)[0, 0, \alpha^2] \)
- \( P_5 = \text{conv}(P_1, P_4)[0, 0, P_3] \)
- \( rt = \text{root}(P_5) \)
- \( f_1 = \text{sqrt}(rt) \)
- \( f_2 = -\text{sqrt}(rt) \)

**Matlab code 2**: \( \alpha = k/(4\pi^2 m) \)
- \( P_1 = [-1, 0, \alpha] \)
- \( P_2 = [-1, 0, 2\alpha] \)
- \( P_3 = [\alpha^2, 0, -2\alpha^3] \)
- \( P_4 = \text{conv}(P_2, P_2)[0, 0, 0, \alpha^2] \)
- \( P_5 = \text{conv}(P_1, P_4)[0, 0, 0, P_3] \)
- \( rt = \text{root}(P_5) \)

**HW**: Let \( k = 4 \times 10^6 \) Newtons/m
\( m = 5000 \) kg

Write a Matlab code to find the roots of the equation.