Examples of Buckling

Problem 1

The structure shown below is built of steel \((E = 200 \text{ GPa})\) beams of square cross section \((b = 30 \text{ mm})\) and is acted upon by force \(P\). The dimensions are \(l = 2 \text{ m}\) and \(d = 1.3 \text{ m}\).

a) Which bar will buckle first?

b) What is the buckling load \(P_{cr}\) of the structure?

\[
\begin{align*}
\frac{P_{BC}}{P_{AB}} &= \frac{\sqrt{l^2 + d^2}}{\frac{\ell}{\ell}} \approx 1.044 \quad \text{and} \quad \frac{P_{crBC}}{P_{crAB}} = \frac{L_{AB}^2}{L_{BC}^2} \approx 3.67 \\
\text{The AB bar will buckle first.}
\end{align*}
\]

\[
P_{AB} = P \frac{\ell}{d}
\]

\[
P_{cr} = \frac{d}{\ell} P_{crAB} = \frac{\pi^2 d E I}{4 \ell^5}, \quad \text{where} \quad I = \frac{b^4}{12}
\]

\[
P_{cr} \approx 9993 \text{ N}
\]
Problem 3
The structure \((\ell = 3 \text{ m})\) shown below is built of steel \((E = 210 \text{ GPa})\) beams of square cross section \((b = 23 \text{ mm})\) and is acted upon by force \(P\). Plot the buckling load of the structure as a function of \(\theta\) for all possible loading directions. What is the lowest buckling load and at what loading direction does it occur?

![Diagram of the structure](image)

**solution:**

\[ P_{crAB} = \frac{\pi^2 EI}{L_{AB}^2} \approx 5,370.4 \text{ N} \quad \text{and} \quad P_{crBC} = \frac{\pi^2 EI}{L_{BC}^2} \approx 2,685.2 \text{ N} \]

\[ P\cos\theta = P_{AB} + \frac{P_{BC}}{\sqrt{2}} \quad \text{and} \quad P\sin\theta = -\frac{P_{BC}}{\sqrt{2}} \]

\[ P = -\frac{P_{BC}}{\sqrt{2}\sin\theta} \quad \text{and} \quad P = \frac{P_{AB}}{\cos\theta + \sin\theta} \]

\[ P_{cr1} = -\frac{P_{crBC}}{\sqrt{2}\sin\theta} \quad \text{if} \quad \sin\theta < 0 \quad \text{and} \quad P_{cr1} = \infty \quad \text{else} \]

\[ P_{cr2} = \frac{P_{crAB}}{\cos\theta + \sin\theta} \quad \text{if} \quad \cos\theta + \sin\theta > 0 \quad \text{and} \quad P_{cr1} = \infty \quad \text{else} \]

\[ P_{cr} = \min\{P_{cr1}, P_{cr2}\} \]
$P_{cr \min} = \frac{P_{crBC}}{\sqrt{2}} = 1,898.7 \text{ N at } \theta = 270^\circ (-90^\circ)$